The *International Conference on Mathematical Modelling in Applied Sciences, ICMMAS'17*, is organized by Peter the Great Saint Petersburg Polytechnic University (SPbPU), Saint Petersburg-Russia, during the period July 24-28, 2017 http://icmmas.alpha-publishing.net/. This conference is aimed to bring experts, researchers and postgraduate students on Mathematical and Computational Modelling in several fields of Science, Technology and Engineering, such as theoretical and computational aspects in Mathematics, Informatics, Physics, Chemistry, Mechanics, Biology, Economics, and other sciences, from the entire world in order to discuss high level scientific questions, exchange solid knowledge of pure and applied sciences, and investigate diverse backgrounds, theoretically and practically.

This meeting is bringing together about 200 internationally known speakers and exhibitors from around the world. The proposed Scientific Program of the conference is including plenary lectures, contributed oral talks, poster sessions and listeners. Some suggested special session/mini-symposium are also provided by the scientific committee. The areas of interest include but are not limited to:

**TOPICS**

- Models Based on Analytical, Numerical and Experimental Solutions
- Mathematical Methods for Physical and Biological Processes
- Ordinary and Partial Differential Equations: Theory and Applications
- Control Theory, Optimization and their Applications
- Probability, Statistics and Numerical Analysis
- Inverse Problems: Modelling and Simulation
- Modern Fractional Dynamic Systems and Applications
- Computational Methods in Sciences and Engineering
- Heat and Mass Transfer in Fractal Medium

ICMMAS'17 is also supported by the following high ranked well known international journals:

1. **Journal of Optimization Theory and Applications**
   - Impact Factor: 1.160
   - Guest Editors: Marc Bonnet, Amar Debbouche and Juan J. Nieto

2. **Mathematical Methods in the Applied Sciences**
   - Impact Factor: 1.002
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   - Impact Factor: 0.987
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PLENARY LECTURES
Degenerate Analytic in a Sector Resolving Operators Family

Vladimir E. Fedorov¹, Elena A. Romanova², Amar Debbouche³

¹,²Chelyabinsk State University, Russia ²Guelma University, Algeria

E-mail: ¹kar@csu.ru; ²linux_21@mail.ru; ³amar_debbouche@yahoo.fr

Abstract We define a class of operators pairs in terms of their relative resolvents. It is shown that for pairs of operators from this class the generalization of resolving operators on the case of degenerate evolution equation with Caputo derivative is exponentially bounded and analytic in a sector. But they forms resolving operators family for the equation only in the case of its weak degeneration. In the case of strong degeneration there is no limit in zero of these operators for integer order equations, and there is no Caputo derivative in every $t > 0$ for fractional order equations.

Introduction

We are concerned with the fractional degenerate evolution equation

$$D_t^\alpha L u(t) = M u(t)$$  \hspace{1cm} (1)

where $U, V$ are Banach spaces, $L : D_L \rightarrow V, M : D_M \rightarrow V$ are densely defined in $U$ linear closed operators, $\ker L \neq \{0\}$, $D_t^\alpha$ is the Caputo fractional derivative with $\alpha > 0$. If operator $L$ is continuously invertible, then equation (1) is equivalent to equation

$$D_t^\alpha v(t) = A v(t)$$  \hspace{1cm} (2)

with the linear closed operator $A = M L^{-1}$ or $A = L^{-1} M$. Sufficient and necessary conditions for the existence of analytic in a sector exponentially bounded resolving operators family of equation (2) can be found in [1][2]. Similar result for equation (1) with a degenerate operator $L$ will be discussed in this work. Note that weakly degenerate case, when the degeneration subspace of equation (1) coincides with ker $L$, differs substantially from the strongly degenerate one with the degeneration subspace containing of $M$-adjoint vectors of the operator $L$ also. This situation does not arise in the case of $\alpha = 1$ [3].

Main results

Let $p \in \mathbb{N}_0$, $\alpha > 0$. A pair of operator $(L, M)$ is said to belong to the class $\mathcal{H}^{\alpha}(\theta_0, a_0, p)$, if

(i) there exist constants $a_0 \geq 0$ and $\theta_0 \in (\pi/2, \pi)$, such that the inclusion $\lambda^a \in \rho^L(M)$ is valid for every $\lambda \in S_{a_0, \theta_0} \equiv \{\mu \in \mathbb{C} : |\arg(\mu - a_0)| < \theta_0, \mu \neq a_0\};$

Keywords: resolving operators family; operator semigroup theory; degenerate evolution equation; fractional order equation; Caputo derivative.

2010 Mathematics Subject Classification : 26A33; 34A08; 35R11; 47D99.
(ii) for all $a > a_0, \theta \in (\pi/2, \theta_0)$ there exists such constant $K(a, \theta) > 0$, that for all $\mu_0, \mu_1, \ldots, \mu_p \in S_{a, \theta}$

$$\max \left\{ \left\| \prod_{k=0}^{p} (\mu^a_k L - M)^{-1} L \right\|_{\mathcal{L}(\mathfrak{U})}, \left\| \prod_{k=0}^{p} L (\mu^a_k L - M)^{-1} \right\|_{\mathcal{L}(\mathfrak{U})} \right\} \leq \frac{K(a, \theta)}{\prod_{k=0}^{p} |\mu^a_k - (\mu_k - a)|}.$$

**Theorem 1.** Let $(L, M) \in \mathcal{H}_a(\theta_0, a_0, p)$. Then for $\alpha > 0$ the operators family

$$U_\alpha(t) = \frac{1}{2\pi i} \int \mu^{\alpha - 1} R^L_{\mu^\alpha}(M) e^{\mu t} d\mu \in \mathcal{L}(\mathfrak{U}) : t > 0 \tag{3}$$

is exponentially bounded and analytic in the sector $\{ t \in \mathbb{C} : |\arg t| < \alpha - \pi/2, t \neq 0 \}.$

Denote $R^L_{\mu^\alpha}(M) = \mathcal{L}^0, \ker L^\mu = \mathcal{Q}^0, \mathcal{L}^1 (\mathcal{Q}^1)$ is the closure of $\text{im} R^L_{\mu^\alpha}(M)$ (im$L^\mu$) in the norm of $\mathcal{L}$, $L_k(M_k)$ is the restriction of $L(M)$ on $D_k \equiv D_L \cap \mathcal{L}_k (D_{M_k} \equiv D_M \cap \mathcal{L}_k), k = 0, 1$.

**Theorem 2.** Let $\mathcal{L}$ and $\mathcal{Q}$ be reflexive Banach spaces, $(L, M) \in \mathcal{H}_a(\theta_0, a_0, p)$. Then

(i) $\mathcal{L} = \mathcal{L}^0 \oplus \mathcal{L}^1, \mathcal{Q} = \mathcal{Q}^0 \oplus \mathcal{Q}^1$;

(ii) the projection $P$ on the subspace $\mathcal{L}^1 (\mathcal{Q}^1)$ along $\mathcal{L}^0 (\mathcal{Q}^0)$ has a form $P = s - \lim_{n \to -\infty} nR^L_{\mu}(M)$ ($Q = s - \lim_{n \to -\infty} nL^\mu_{\mu_n}(M)$);

(iii) $L_0, M_0 \in \mathcal{C} \mathcal{L}(\mathcal{L}^0; \mathcal{Q}^0), L_1, M_1 \in \mathcal{C} \mathcal{L}(\mathcal{L}^1; \mathcal{Q}^1)$

(v) there exist $L_1^{-1} \in \mathcal{C} \mathcal{L}(\mathcal{Q}^1; \mathcal{L}^1), M_0^{-1} \in \mathcal{L}(\mathcal{L}^0; \mathcal{Q}^0)$;

(vi) if $p = 0$ [4], or $\alpha = 1$ [3], then for $t > 0 U_\alpha(t) = U_\alpha^1(t) P = PU_\alpha^1(t)$ and the operators [3] with $U_\alpha(0) = P$ forms the resolving operators family for [1];

(vii) if $p \in \mathbb{N}, \alpha \in \mathbb{N} \setminus \{1\}$, then for $t > 0 U_\alpha(t) = U_\alpha^1(t) P = PU_\alpha^1(t)$ and for $u_0 \in D_M U_\alpha(t) u_0$ is a solution of [1] at every $t > 0$;

(viii) if $p \in \mathbb{N}, \alpha \in \mathbb{R}_+ \setminus \mathbb{N}$, then for $t > 0$

$$U_\alpha(t) = U_\alpha^1(t) P + \sum_{k=1}^{p} \frac{(M_0^{-1} L_0)^k (I - P)}{\Gamma(1 - \alpha k) \Gamma \alpha k}, \quad U_\alpha[\ker L] = \{0\},$$

and for $u_0 \in \mathcal{L}^1 \setminus \ker L$ the Caputo derivative $D^\alpha_t LU_\alpha(t) u_0$ is not defined.

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Applicational aspects of a hypercomplex operator theory

Wolfgang Sproessig

Freiberg University of Mining and Technology, Germany

E-mail: sproessig@math.tu-freiberg.de

Abstract  Hypercomplex analysis can be seen as some kind of "complex function theory" for higher dimensions, where complex numbers are replaced by quaternions, coquaternions, split quaternions, Clifford numbers, octonions, sedenions etc.. Hyperholomorphic functions play the role of holomorphic functions of the complex function theory in the plane. They are zero solutions of higher-dimensional versions of Cauchy-Riemann equations (Riesz system, Fueter system, system of Moisil-Teodorescu, etc.). In this talk we reduce our considerations to quaternion valued functions over 3D-domains. As in the classical function theory also in higher dimensional versions some operators are important: Dirac operator, Teodorescu transform, Cauchy-Fueter operator as well as the orthoprojections on the Bergman space of the Hilbert space (module) and on its complement. For boundary value problems we also need so-called projections of Plemelj type which are connected with the Cauchy-Fueter operator. We use and derive analoga of basic theorems of the plane function theory. Using Bergman-Hodge decompositions boundary value problems can be considered. In this talk will be magnetic fluid flow and shallow water problems in the focus of applications. Further results belonging to Appell polynomials, hypercomplex parabolic Dirac operators, Dunkl operators, generalized Eisenstein series, Clifford transform analysis, spaces of holomorphic functions, etc.. are shortly explained.

References


Keywords: hypercomplex analysis; fluid flows; quaternions; transform analysis.

2010 Mathematics Subject Classification: 30GXX; 33CXX; 76-XX; 35K-XX.
About some new derivatives in the area of fractional calculus and its applications

Dumitru Baleanu1,2

1 Cankaya University, Department of Mathematics, 06530, Balgat, Ankara, Turkey
2 Institute of Space Sciences, Magurele-Bucharest, Romania

E-mail: dumitru@cankaya.edu.tr

Abstract In my talk I will present the properties and the applications of some newly introduced fractional new derivatives.

Introduction

During the last few decades, fractional calculus was applied in the study of so-called anomalous social and physical behaviors, where scaling power law of fractional order appears universal as an empirical description of such complex phenomena [1, 2, 3]. We recall that the standard mathematical models of integer-order derivatives, including nonlinear models, do not work adequately in many cases where power law is clearly observed. To accurately describe the non-local, frequency- and history-dependent properties of power law phenomena, some alternative modeling tools have to be introduced such as fractional calculus and its generalizations.

In this lecture I will review some new fractional derivatives introduced recently within the fractional calculus area. Some real world applications will be given as well.

References


Keywords: fractional calculus; fractional derivatives; fractional integrals; Mittag-Leffler function.
2010 Mathematics Subject Classification: 26A33


Noether’s Symmetries in Quantum Cosmology

Salvatore Capozziello

Università di Napoli "Federico II" and INFN Sez. di Napoli

E-mail: capozzielo@na.infn.it

Abstract We discuss the Hamiltonian dynamics for cosmologies coming from theories of gravity. In particular, minisuperspace models are taken into account searching for Noether symmetries. The existence of conserved quantities gives selection rules to recover classical behaviors in cosmic evolution according to the so called Hartle criterion, that allows to select correlated regions in the configuration space of dynamical variables. We show that such a statement works for general classes of Theories of Gravity and is conformally preserved. Furthermore, the presence of Noether symmetries allows a straightforward classification of singularities that represent the points where the symmetry is broken. Examples for nonminimally coupled and higher-order models are discussed.

References


Keywords: Noether’s symmetries; dynamical systems; cosmology

2010 Mathematics Subject Classification: 35L65; 35F21; 37C80
Modeling, global stability and optimal control of HIV/AIDS through PrEP

Cristiana J. Silva\(^1\), Delfim F. M. Torres\(^2\)

\(^1,2\) CIDMA, DMat, University of Aveiro, Portugal

E-mail: \(^1\)cjoaosilva@ua.pt; \(^2\)delfim@ua.pt

Abstract  Pre-exposure prophylaxis (PrEP) consists in the use of an antiretroviral medication to prevent the acquisition of HIV infection by uninfected individuals and has recently demonstrated to be highly efficacious for HIV prevention. We propose a new mathematical epidemiological model for HIV/AIDS transmission including PrEP, which generalizes the HIV/AIDS submodel that we previously proposed in [1].

We begin by considering the particular case with no PrEP and study this mathematical model for the transmission dynamics of the human immunodeficiency virus (HIV). Global stability of the unique endemic equilibrium is proved. Then, based on data provided by the “Progress Report on the AIDS response in Cape Verde 2015”, we calibrate the model to the cumulative cases of infection by HIV and AIDS from 1987 to 2014 and we show that our model predicts well such reality. Finally, a sensitivity analysis is done for the case study in Cape Verde. We conclude that the goal of the United Nations to end the AIDS epidemic by 2030 is a nontrivial task [2].

Then, we prove the existence and global stability of the disease free equilibrium for the full HIV/AIDS-PrEP model, the existence of an unique endemic equilibrium and its global stability for some specific cases. An optimal control problem with mixed state control constraint is proposed and analyzed, where the control function represents the PrEP strategy and the mixed constraint models the fact that, due to PrEP costs, epidemic context and program coverage, the number of individuals under PrEP is limited, at each instant of time, for a fixed interval of time. The objective of the optimal control problem is to determine the PrEP strategy that satisfies the mixed state control constraint and minimizes the number of individuals with pre-AIDS HIV-infection as well as the costs associated with PrEP. The optimal control problem is solved analytically. Through numerical simulations, we demonstrate that PrEP reduces HIV transmission [3].

Acknowledgments

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Keywords: mathematical modeling of HIV/AIDS, antiretroviral therapy (ART), PrEP, global stability, United Nations UNAIDS strategy, optimal control, Cape Verde.

2010 Mathematics Subject Classification: Primary: 34C60, 92D30; Secondary: 34D23, 49K15.
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Neural Network Modeling of Complex Systems Described by Differential Equations and Other Data

Alexander Vasilyev¹, Dmitry Tarkhov², Valery Antonov³

Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: ¹a.n.vasilyev@gmail.com; ²dtarkhov@gmail.com; ³antonovvi@mail.ru

Abstract We propose an original approach to the construction of the approximate parametric neural network models of some complex systems in the case of heterogeneous information – differential equations, partially specified initial-boundary conditions and other ones, experimental data, etc. We consider the fundamental modifications of the classical numerical methods in the form of multilayer model technique. These approaches were tested on several different tasks.

Introduction

The modern direction of information modeling – Neural network modeling – was successfully developed by SPbPU professors A. Vasilyev and D. Tarkhov since 2002. Neural networks [S. Haykin] – the known class of mathematical models commonly applied to data processing, pattern recognition, control of complex systems, etc. In our opinion, the reasons for the successful application of neural networks lie in their useful properties: 1) The possibility of constructing models of arbitrary complexity by the standard rules from a finite set of standard elements; 2) The property of universal approximator – functions from a broad class (e.g., continuous ones) can be arbitrarily accurately approximated by neural networks; 3) Resistance to errors in the input data and miscalculations. Only the main properties of neural networks are noted here; along with them, it is worth noting the smoothness and the behavior at infinity of the base neuroelements, and inherent in neural networks natural parallelism, which is undoubtedly useful and effective for supercomputer calculations.

Main results

In 2002 A. Vasilyev and D. Tarkhov became interested in the possibility of applying neural networks to approximate solution of differential equations (ODEs and PDEs). The result was a new scientific direction, in its scope and prospects comparable to the finite element method. A few tens of tasks were successfully resolved – as model problems, and with practical interest. Among the tasks – the problems in the composite areas, with fixed, required and customizable borders, incorrect or ill-posed problems.
stiff problems, etc [1]. The uniform process of building the neural network models (not only neural ones) was developed. We see two things as the main reasons for the success.

The first circumstance is the new perspective on mathematical modeling. A classical approach to mathematical modeling consists of two stages. At the first stage, on the known information about the physical processes occurring in the simulated system, one builds a model of the system in the form of (integro-)differential equations, boundary, and other conditions. At the second stage, one searches for the numerical solution of the constructed equations. Thus differential equations and conditions are accepted as the object of modeling, and the quality of the approximate solution is evaluated by proximity to the exact solution of the corresponding boundary value problem. How close the exact solution describes the processes in a real system, this usually remains beyond consideration. Another point of view, which we follow, is to consider equations, boundary conditions, observations, etc. as some approximate heterogeneous information about the system. It seems to be right considering the hierarchy of models with different accuracy (and range of applicability); these models can be updated with a new piece of information. The process of determining the model parameters (training neural networks) is the process of finding a global extremum of an error functional.

The second circumstance is that the authors in the new approach moved away from commonly used in solving such kind of problems collocation method, consisting in the fact that the accuracy to satisfy the differential equation in some region is characterized by the error in the fixed point set of the region. Our approach consists in the regeneration of a set of points after a few steps of the optimization process for the previously mentioned error functional. In the end, we get the optimization process not specific for one functional, but for a series of functionals. This approach allows avoiding the problem of sticking of the optimization process in local minima and the situation when the error of the equation satisfaction is small at the collocation points and large at other points of the considered region.

The next important advance in a new area of science was to develop methods for construction of parametric neural network models. When modeling real systems it is often the case when some parameters of the object are known imprecisely or they are subject to selection in the modeling process. Our approach allows building models that include these parameters among the input variables. The model-building process differs only in the additional regeneration of parameters in the process of test point regeneration.

The neural network models typically have a multilayer structure. In 2016, the authors proposed a methodology for constructing multilayer models [2]. The essence of such a technique consists in applying the classical methods of Euler, Runge-Kutta, etc. not to a constant interval, but to the interval with a variable upper limit. The advantages of this approach are as follows: 1) In contrast to classical methods, the result is not a set of numbers, but a function, and the parameters of the problem are in the set of function arguments. This function can be improved by methods described earlier; 2) Given the fact that the functions contained in the problem statement can be arbitrarily accurately approximated by neural networks, we obtain as a result of this construction a multilayer neural network solution to arbitrarily high accuracy without resource-intensive learning procedure; 3) To accelerate the computation process, the similar network can be realized in hardware by specialized neurochip. Such neural chips can be manufactured for solving the most common standard problems. Note that the construction of such multilayer neural network solutions can serve as an analog (version) of deep learning.
References


On multi-agent systems on time scales

Agnieszka B. Malinowska¹, E. Girejko²

¹,² Bialystok University of Technology, Poland

E-mail: ¹a.malinowska@pb.edu.pl; ²e.girejko@pb.edu.pl

Abstract  The lider-following consensus problem of multi-agents systems on time scales is considered. First, a consensus control law with a time-varying state is given. The theory of dynamical equations on time scales is used to study the stability of systems. Then, we look for optimal control strategies for the multi-agent system to attain consensus in such a way that the control mechanism is included in the leader dynamics. Necessary optimality conditions are obtained by the use of the weak maximum principle for control problems on time scales. Several simulations are presented to verify the theoretical results.

Introduction

Recently, the consensus problem of multi-agent systems has attracted a lot of attention by many researcher from a wide range of disciplines, such as biology [1], physics [5], robotics and control engineering [2]. In the consensus problem the goal is for the individual nodes (agents) to reach an agreement on the states of all agents. Consensus algorithms are based on nearest-neighbour rules [4], bounded confidence [3] or a virtual leader [6]. In this work, we consider multi-agent systems with a leader on time scales. The advantage of using the time scales theory lies in the fact that one does not need to consider continuous-time and discrete-time cases separately, those systems are two particular cases of time scale multi-agent systems. Moreover, the time scale theory allows us to consider hybrid systems that evolve continuous with discrete-time events or systems with nonuniform sampling periods.

Main results

Problem I. Consider the multiagent system consisting of $n$ agents from the set $N = \{1, \ldots, n\}$ on a time scale $\mathbb{T}$. The single-integrator dynamics of each agent is given by

$$x^A_i(t) = u_i(t), \quad i \in N,$$

(4)

where $x_i : \mathbb{T} \to \mathbb{R}$ is the state function for the $i$th agent, $u_i : \mathbb{T} \to \mathbb{R}$ is the control input function for the $i$th agent. The virtual leader for multiagent system [4] is an isolated agent described by

$$x^A_0(t) = f(t),$$

(5)

Keywords: multi-agent systems; consensus; time scales; stability; optimal control.

2010 Mathematics Subject Classification: 39A12; 34N05; 49K05; 26E70
where $f(t)$ is the nonlinear dynamics of the virtual leader for $f$ being $rd$-continuous function. Using only the local interaction between agents we propose the consensus control law of tracking the virtual leader as follows:

$$u_i(t) := f(t) - \beta \left[ \sum_{j=1}^{n} a_{ij} (x_i(t) - x_j(t)) + b_i (x_i(t) - x_0(t)) \right], \quad (6)$$

where $a_{ij}$ ($i, j = 1, 2, \ldots, n$) is the $(i, j)$th entry of the adjacency matrix $A$, $b_i = 1$ if the dynamics of the virtual leader is available to agent $i$ and $b_i = 0$ otherwise, $\beta > 0$ is a constant parameter.

We state and prove conditions under which control law (6) solves the consensus problem for multi-agent system (4)–(5).

Problem II. Consider the optimal control problem:

$$\min J(x, u)$$

$$:= \frac{\alpha}{2} \left( x_0(T) - x_d(T) \right)^2 + \frac{1}{2 n^2} \int_0^T \sum_{i,j=1}^{n} (x_i(t) - x_j(t))^2 \Delta t + \frac{1}{2} \int_0^T \sum_{i=1}^{n} (x_0(t) - x_i(t))^2 \Delta t + \nu \int_0^T (u(t))^2 \Delta t, \quad (7)$$

subject to

$$\begin{cases}
  x_0^\Delta(t) = u(t) \\
  x_i^\Delta(t) = -\left( \sum_{j=1}^{n} a_{ij} (x_i(t) - x_j(t)) + b_i (x_i(t) - x_0(t)) \right), \quad i \in N
\end{cases} \quad (8)$$

with given initial positions $x_i(0) \in \mathbb{T}$, for $i = 0, 1, \ldots, n$ and $x \in C_{prd}[a, b|_\mathbb{T}$ and $u \in C_{prd}[a, \rho(b)|_\mathbb{T}$. In the cost functional $\alpha$ and $\nu$ are weight constants, the second and the third terms are the energy of the system. Additionally, the tracking functional at final time requires the leader to approach a desired target configuration $x_d$. The last term in (7) represents the cost of the control.

We state and prove necessary optimality conditions for problem (7)–(8).

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References


Robust Statistics: A Brief Review of Ideas and Tools with Their Applications to Signal Processing

Georgy L. Shevlyakov

Peter the Great St. Petersburg Polytechnic University, Russia
Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, Russia

E-mail: gshevlyakov@yahoo.com

Abstract Main approaches to robust statistics, namely Huber's minimax and Hampel's approach based on influence functions, are considered together with the corresponding methods. Several examples of their application to the solution of conventional data analysis and signal processing problems are enlisted.

Introduction

Robust statistics as a branch of mathematical statistics appeared due to the seminal works of Tukey (1960), Huber (1964), and Hampel (1968). It has been intensively developed since the sixties of the last century and is definitely formed by present.

The principal reason of research in this field of statistics is of a general mathematical nature. Optimality (accuracy) and stability (reliability) are the mutually complementary characteristics of many mathematical procedures. It is well-known that the performance of optimal procedures is, as a rule, rather sensitive to 'small' perturbations of prior assumptions. In mathematical statistics, the classical example of such unstable optimal procedure is given by the least squares method: its performance may become disastrously poor under small deviations from normality.

Roughly speaking, robustness means stability of statistical inference under the departures from the accepted distribution models.

Main approaches and ideas

The first general approach to robustness is based on the minimax principle (Huber, 1964). The minimax approach aims at the worst situation for which it suggests the best solution. Thus, in some sense, this approach provides a guaranteed result, perhaps too pessimistic. However, being applied to the problem of parameter estimation, it yields a robust modification of the principle of maximum likelihood.

The main advantage of robust methods is their lower sensitivity to possible variations of data distributions as compared to conventional statistical methods. Thus it is necessary to have specific mathematical

Keywords: robustness; statistical methods; minimax principle, influence function tools, robust signal processing, robust exploratory data analysis, robust power spectrum estimation.

2010 Mathematics Subject Classification: 62F35; 62M10; 62M15.
tools that allow to analyze the sensitivity of estimates to outliers, rounding-off errors, etc. On the other hand, such tools make it possible to solve the inverse problem: to design estimates with the required sensitivity.

The second general approach to robustness is based on the the above-mentioned apparatus, namely, the sensitivity curves and the influence functions (Hampel et al., 1986).

All those robust methods are developed for solving the conventional statistical problems of estimation of location, scale, correlation, regression, etc. (Huber, 1981; Hampel et al., 1986; Shevlyakov and Vilchevski, 2002, 2011; Shevlyakov and Oja, 2016).

Applications and open problems

The developed robust methods are applied to various problems of statistical data and signal analysis: namely, to exploratory data analysis and its technologies, mostly treating an important problem of detection of outliers in the data; to robust estimation of time series power spectra, to the problems of robust signal detection.

Finally, several open problems in multivariate statistical analysis are outlined.

References


F-transform as a Technique for Boundary Value Problems: Ordinary and Fuzzy

Irina Perfilieva

University of Ostrava, IRAFM, Czech Republic

E-mail: Irina.Perfilieva@osu.cz

Abstract In this contribution, we discuss numerical solutions of various BVPs with ordinary and fuzzy boundary values by the method of F-transform. We analyze linear and non-linear second-order BVPs and demonstrate the advantages of the F-transform method in comparison with its classical counterparts. We argue that this technique increases robustness of the numerical approach, while keeping the expected accuracy.

Introduction

The F-transform-based approach to ordinary differential equations (ODE) has been initiated in [1] where a particular initial value problem was solved and a certain generalization of the Euler method was developed. The effectiveness and potential of the proposed method in comparison with classical techniques was discussed. Later, the same approach has been successfully used in [2] for a second order initial value problem. In [3], the second order Runge-Kutta method was generalized and outperformed by using the F-transform components instead of functional values.

The boundary value problem (BVP) for a linear second-order ODE has been considered in [5] where one particular case of this BVP was efficiently solved. In [6], the study was continued and the estimation of the accuracy order of a particular numerical solution was obtained.

In this contribution we are focused on the following particular BVPs. The first considered problem is a two-point boundary value problem for a second-order differential equation:

\[ y'' = f(x, y, y'), \quad x \in (a, b), \]  
\[ r_1(y(a), y'(a)) = 0, \quad r_2(y(b), y'(b)) = 0, \]  

where \( f = f(x, u, v) \), \( r_1 = r_1(u, v) \) and \( r_2 = r_2(u, v) \) are given continuous functions on \([a, b]\).

The differential equation of second considered problem is a particular (linear) case of [5]. Different to [10], boundary values are characterized by fuzzy numbers:

\[ y'' + d_1 y' + d_0 y = f(x), \quad x \in [x_1, x_n], \]  
\[ y(x_1) = \tilde{Y}_1, \quad y(x_n) = \tilde{Y}_n, \]  

In [11], \( d_1, d_0 \) are reals and \( \tilde{Y}_1, \tilde{Y}_n \) are boundary values represented by unimodular fuzzy sets.

Keywords: F-transform; second order differential equation; boundary value problem; fuzzy boundary values.

2010 Mathematics Subject Classification: 47B32; 46E22; 39A10; 65K15.
Main results

For the two considered BVPs, we propose new numerical schemes that are based on the modern technique of F-transforms. A new modification of the shooting method based on the second degree F-transform is proposed for the BVP (9)-(10). We demonstrate the effectiveness of the proposed method in the class of second order numerical methods. For this purpose we choose the second order Runge-Kutta method. In more details, both methods use the same shooting scheme, but differ in the solution of the corresponding IVP. We applied both methods (with the same initialization parameters) to all selected test equations and made sure of the higher accuracy of the F-transform-based shooting. Moreover, the stability of the proposed method and its ability to "ignore noise" was verified on examples.

For a BVP (11) with fuzzy boundary values, we define a solution in the form of fuzzy relation and discuss how the approximate solution can be obtained. We split (11) into three ordinary BVPs with the same second-order ODE. We impose restrictions on $d_1, d_0$ that guarantee the solvability of the corresponding three ordinary BVPs. Then, we apply the F-transform to both sides of these ordinary BVPs and obtain numerical solutions in terms of the F-transform components. The latter are used for definition of the corresponding system of fuzzy relation equations which is formulated in the language of Łukasiewicz operations on $[0, 1]$. We prove that under made assumptions this system is solvable. We characterize the set of solutions and analyze their approximation quality. Finally, we present the illustrative example.

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References


TALKS
Application of fixed point theory and study of the existence of solutions of nonlinear coupled differential systems

Imran Talib\(^1\), Naseer Ahmad Asif\(^2\), Cemil Tunc\(^3\)

\(^1\)Department of Mathematics, Virtual University of Pakistan.
\(^2\)Department of Mathematics, School of Science, University of Management and Technology, Lahore, Pakistan.
\(^3\)Department of Mathematics, Faculty of Sciences, Yuzuncu Yil University, Vain, Turkey.

E-mail: \(^1\)imrantaalib@gmail.com; \(^2\)naseerasif@yahoo.com; \(^3\)cmtunc@yahoo.com

Abstract In this talk, we elaborate the application of fixed point theorems by investigating the existence of solutions of certain nonlinear second-order coupled boundary value problems with dependence on the first order ordinary derivatives of nonlinear functions applying coupled lower and upper solutions approach. The existence results are constructed with the help of Schauder's fixed point theorem and Arzelà-Ascoli theorem. Our work is an extension of the works presented in the literature. The very important and considerable impact of our research is that we give a general approach of existence of solutions to cover various systems of boundary value problems in a unified way, which avoids treating these problems on a case-by-case basis. The applicability of the developed theoretical results is ensured by considering some examples.

Introduction

Problems with coupled boundary conditions (BCs) have been focus of many studies not only because of a theoretical interest but also they have rich applications in mathematical biology, chemical systems, engineering, life sciences, and many more, see for example [1, 2, 3] and references therein. Motivated by the vast applications of the problems with coupled BCs and to fill some gaps in the literature, we are encouraged to develop the existence result for the following nonlinear coupled system of ordinary differential equations subject to nonlinear coupled BCs:

\[
\begin{align*}
-u''(t) &= f_1(t,u(t),v(t),u'(t),v'(t)), \quad t \in [0,1], \\
-v''(t) &= f_2(t,u(t),v(t),u'(t),v'(t)), \quad t \in [0,1], \\
\sigma(u(0),v(0),u(1),v(1),u'(0),v'(0)) &= (0,0), \\
\rho(u(0),v(0),u(1),v(1),u'(1),v'(1)) &= (0,0),
\end{align*}
\]

where \(f_1, f_2 : [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}\), and \(\sigma, \rho : \mathbb{R}^6 \to \mathbb{R}^2\) are continuous functions.

Keywords: Lower and upper solutions; Nonlinear coupled system; Nonlinear coupled boundary conditions; Arzela-Ascoli theorem; Schauder's fixed point theorem.

2010 Mathematics Subject Classification: 34A12; 37C25; 34C15.
Main results

Lemma 3. Let \( \hat{D} : C^2[0,1] \times C^2[0,1] \to C_0[0,1] \times C_0[0,1] \times \mathbb{R}^2 \times \mathbb{R}^2 \) be defined by

\[
[\hat{D}(u, v)](t) = \left[ u'(t) - u'(0) - \psi \int_0^t u(s) \, ds, v'(t) - v'(0) - \psi \int_0^t v(s) \, ds, \right.
\]
\[
\left. (\lambda_1 u(0) + \lambda_2 u(1), \lambda_3 v(0) + \lambda_4 v(1), (\lambda_1' u(0) + \lambda_2' u(1), \lambda_3' v(0) + \lambda_4' v(1)) \right),
\]

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_1', \lambda_2', \lambda_3', \lambda_4' \in \mathbb{R} \), with \( \psi > 0 \), such that

\[
(\lambda_1 \lambda_4 - \lambda_2 \lambda_3)(\lambda_1' \lambda_4' - \lambda_2' \lambda_3') \left( e^{-\sqrt{\psi}} - e^{\sqrt{\psi}} \right) \neq 0,
\]

and here

\[
C_0[0,1] = \{ x \in C^1[0,1] : x(0) = 0 \}.
\]

Then \( \hat{D}^{-1} \) exists and is continuous and defined by

\[
[\hat{D}^{-1}(w, z, \xi, \eta, \nu, \rho)] = \left( \chi_1 e^{\sqrt{\psi} t} + \chi_2 e^{-\sqrt{\psi} t} + \frac{1}{2} \int_0^t e^{\sqrt{\psi}(s-t)} w(s) \, ds \right.
\]
\[
+ \frac{1}{2} \int_0^t e^{\sqrt{\psi}(s-t)} z(s) \, ds + \frac{1}{2} \int_0^t e^{\sqrt{\psi}(s-t)} \xi(s) \, ds,
\]

Theorem 4. Let system \([12] - [13]\) having \((\alpha_1, \alpha_2), (\beta_1, \beta_2)\) coupled lower and upper solutions respectively and the nonlinear functions \( f_1 \) and \( f_2 \) relative to the intervals \([\alpha_1(t), \beta_1(t)]\) and \([\alpha_2(t), \beta_2(t)]\) satisfy the Nagumo assumptions respectively. Moreover, assume that \( \varphi \) and \( \sigma \) are nonincreasing and nondecreasing in the fifth and sixth arguments respectively and

\[
\varphi_a(x, y) := \varphi(x, y, \alpha_1(1), \alpha_2(1), \alpha_1'(1), \alpha_2'(1)),
\]
\[
\varphi_b(x, y) := \varphi(x, y, \beta_1(1), \beta_2(1), \beta_1'(1), \beta_2'(1)),
\]

are monotone on \([\alpha_1(0), \beta_1(0)] \times [\alpha_2(0), \beta_2(0)]\) and

\[
\sigma_a(x, y) := \sigma(x, y, \alpha_1(0), \alpha_2(0), x, y, \alpha_1'(0), \alpha_2'(0)),
\]
\[
\sigma_b(x, y) := \sigma(x, y, \beta_1(0), \beta_2(0), x, y, \beta_1'(0), \beta_2'(0)),
\]

are monotone on \([\alpha_1(1), \beta_1(1)] \times [\alpha_2(1), \beta_2(1)]\).

Then system \([12] - [13]\) have at least one solution \((u, v) \in \left[ \alpha_1, \beta_1 \right] \times \left[ \alpha_2, \beta_2 \right] \) and \((- \mathcal{N}_1, - \mathcal{N}_2) \leq (u'(t), v'(t)) \leq (\mathcal{N}_1, \mathcal{N}_2), \ t \in [0, 1] \).

References


Some properties of \((p, q)\)-orthogonal polynomials

Iván Area\(^1\), Mohammad Masjed-Jamei\(^2\), Fatemeh Soleyman\(^3\)

\(^1\)Departamento de Matemática Aplicada II, E.E. Aeronaútica e do Espazo, Universidade de Vigo, Campus As Lagoas s/n, 32004 Ourense, Spain
\(^2\)K.N.Toosi University of Technology, Department of Mathematics, P.O. Box 16315–1618, Tehran, Iran
\(^3\)K.N.Toosi University of Technology, Department of Mathematics, P.O. Box 16315–1618, Tehran, Iran

E-mail: \(^1\)area@uvigo.es; \(^2\)mmjamei@kntu.ac.ir; \(^3\)fsoleyman@mail.kntu.ac.ir

Abstract In this talk, we introduce \((p, q)\)-Sturm-Liouville difference equations and prove that the \((p, q)\)-difference of a solution of the equation is also solution of an equation of the same type (hypergeometric condition). From this property a Rodrigues-type formula is derived. From the Bochner type characterization we derive the three-term recurrence relation coefficients for these polynomials. Moreover, structure relation and \((p, q)\)-difference representation are also obtained. Finally, some specific examples are studied in detail, giving \((p, q)\)-analogues of classical shifted Jacobi, Laguerre, and Hermite polynomials. Limit transitions from these \((p, q)\)-analogues to the classical families are also given.

Acknowledgments

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References


Keywords: orthogonal polynomials; \((p, q)\)-difference operator; limit transitions

2010 Mathematics Subject Classification: 34B24; 39A70; 33C47.


The impact of pre-exposure prophylaxis (PrEP) and screening on the dynamics of HIV

Carla M.A. Pinto\textsuperscript{1}, Ana R. M. Carvalho\textsuperscript{2}

\textsuperscript{1}School of Engineering, Polytechnic of Porto \hspace{1em} \textsuperscript{2}University of Porto, Portugal

E-mail: \textsuperscript{1}cap@isep.ipp.pt; \textsuperscript{2}up200802541@fc.up.pt

Abstract We analyse the impact of pre-exposure prophylaxis (PrEP) and screening effects on HIV dynamics in infected patients. Our model incorporates condom use, the number of sexual partners, and treatment for HIV. Numerical simulations are performed and the model is fitted to data on HIV prevalence in Portugal. Moreover, critical epidemiological parameters are also varied. Inferences are made concerning the epidemiological consequences of the model's predictions on public HIV/AIDS planning.

Acknowledgments

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References


Keywords: PrEP, screening, HIV/AIDS, treatment, condom use

2010 Mathematics Subject Classification: 34G25; 92B05.
Some new results for fuzzy differential equations

Alireza Khastan

Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan, Iran

E-mail: khastan@gmail.com

Abstract  In this paper, we study fuzzy differential equations with generalized Hukuhara differentiability. In special case, we investigate the properties of solutions for linear first order fuzzy differential equations. Some examples are provided to illustrate our results.

References


Keywords  Fuzzy differential equations, generalized differentiability, Hukuhara derivative, generalized Hukuhara differentiability

2010 Mathematics Subject Classification  34A07;34A12;03E72.
Some Open Problems in the Reproducing Kernel Method

Ali Akgül¹, Esra Karatas Akgül²

¹Siirt University, Turkey ²Canakkale Onsekiz Mart University, Turkey

E-mail: ¹aliakgul00727@gmail.com; ²esrakaratas@comu.edu.tr

Abstract  This work is devoted to some open problems for reproducing kernel method. The main idea of this work is to construct novel reproducing kernel spaces. We define the open problems and solve some of them in these spaces. We give some applications of this method. Numerical experiments are provided to illustrate the performance of the method.

Introduction

Reproducing kernel space is a special Hilbert space. In recent years, there are many papers on the solution of the nonlinear problems with reproducing kernel method. The concept of reproducing kernel can be traced back to the paper of Zaremba in 1908. It was proposed for discussing the boundary value problems of the harmonic functions. This is the first reproducing kernel corresponding to function family introduced in special case and with the reproducibility proved. In the early development stage of the reproducing kernel theory, most of the works were applied by Bergman. Bergman put forward the corresponding kernels of the harmonic functions with one or several variables, and the corresponding kernel of the analytic function in squared metric, and applied them in the research of the boundary value problem of the elliptic partial differential equation. This is the first stage in the development history of reproducing kernel [1].

The second development stage of the reproducing kernel theory was started by Mercer. Mercer discovered that the continuous kernel of the positive definite integral equation has the positive definite property:

\[ \sum_{i,j=1}^{n} k(x_i, y_j) \zeta_i \zeta_j \geq 0. \]

He named the kernel with this property as positive definite Hermite matrix. He also found out that the positive defined Hermite matrix corresponded to a function family, proposed a Hilbert space with inner product \( \langle f, g \rangle \), and proved the reproducibility of the kernel in this space:

\[ u(y) = \langle u(x), k(x, y) \rangle. \]

The third development stage of the reproducing kernel theory related to Aronszajn. In 1950, he reduced the works of the formers and studied a systematic reproducing kernel theory including the Bergman kernel function. Reproducing kernel method (RKM), which computes the numerical solution, is of great attention to many branches of applied sciences. Many studies have been dedicated to the application of RKM to a wide class of differential equations. The accuracy and power of the method were considered by researches to investigate many applications. Geng and Cui implemented the RKM to handle second-order boundary value problems [2]. Yao and Wang et al. considered a class of singular boundary value problems by the RKM. Zhou et al. [3] applied the RKM successfully to

Keywords: reproducing kernel method; some open problems; kernel functions.

2010 Mathematics Subject Classification: 46E22; 47B32; 74S30.
solve second-order boundary value problems. The RKM was applied to nonlinear infinite-delay-differential equations. Wang and Chao, Zhou and Cui independently employed the RKM to variable-coefficient partial differential equations. Geng and Cui, Du and Cui considered the forced Duffing equation with integral boundary conditions by joining the HPM and RKM. Lv and Cui introduced a novel procedure to solve linear fifth-order boundary value problems. Cui and Du acquired the analytical solution for nonlinear Volterra-Fredholm integral equations by RKM. Wu and Li implemented an iterative reproducing kernel method to get the analytical approximate solution of a nonlinear oscillator with discontinuities [4].

**Main results**

We define some open problems of reproducing kernel method in this work. We obtain very useful theorems and proofs for these problems. We investigate the existence and uniqueness of the solution in the reproducing kernel Hilbert space.

**References**


A Novel Method for the Solution of Blasius Equation in Semi-Infinite Domains

Ali Akgül¹, Esra Karatas Akgül²

1Siirt University, Turkey  2Canakkale Onsekiz Mart University, Turkey

E-mail: ¹aliakgul00727@gmail.com; ²esrakaratas@comu.edu.tr

Abstract  Many known methods fail in the attempt to get analytic solutions of Blasius-type equations. In this work, we apply the reproducing kernel method for investigating Blasius equations with two different boundary conditions in semi-infinite domains. Convergence analysis of the reproducing kernel method is given. The numerical approximations are presented and compared with some other techniques, Howarth's numerical solution and Runge-Kutta Fehlberg method.

Introduction

Nonlinear differential equations are extensive in science and technology. However, finding analytical solutions for this class of equations always has been a challenging work. Many approximate methods were introduced for the analytical solution of nonlinear differential equations in the recent years. Among these, Homotopy Analysis Method (HAM), Adomian Decomposition Method (ADM), Variational Iteration Method (VIM), Differential Transformation Method (DTM), and Homotopy Perturbation Method (HPM) can be referred. Some new techniques for approximate solution of nonlinear differential equations shown up recently, such as Optimal Homotopy Asymptotic Method (OHAM) and Generalized Homotopy Method (GHM) [1, 2].

In the present paper, reproducing kernel method (RKM) has been applied for the solution of two different forms of nonlinear Blasius equation in a semi-infinite domain. The theory of reproducing kernels was introduced by S. Zaremba. Much notice has been given to the work of the RKM to solve many works.

We present two forms of the Blasius equation arising in fluid flow inside the velocity boundary layer as follows.

The first form of the Blasius equation is given as:

\[
\begin{align*}
\frac{u^{(3)}(t) + u(t)u''(t)}{2} &= 0, & 0 \leq t \leq \infty, \\
u(0) &= u'(0) = 0, & u'(t) = 1 \text{ as } t \to \infty.
\end{align*}
\]

The second form is given as:

\[
\begin{align*}
\frac{u^{(3)}(t) + u(t)u''(t)}{2} &= 0, & 0 \leq t \leq \infty, \\
u(0) = 0, & u'(0) = 1, & u'(t) = 0 \text{ as } t \to \infty.
\end{align*}
\]

These equations are the same except for boundary conditions. The first form of the equation is the well-known classical Blasius first derived by Blasius and dates back about a century, which defines the velocity profile of two-dimensional viscous laminar flow over a finite flat plate. This form of the Blasius equation is the simplest form and

Keywords: reproducing kernel functions; Blasius equation.
2010 Mathematics Subject Classification: 46E22; 47B32; 74S30.
the origin of all boundary layer equations in fluid mechanics. The second form of the equation, presented more recently, arises in the steady free convection about a vertical flat plate embedded in a saturated porous medium, Laminar boundary layers at the interface of cocurrent parallel streams, or the flow near the leading edge of a very long, steadily operating conveyor belt [3][4].

Many analytical techniques were introduced to investigate Blasius equation. Comparison with Howarth’s numerical solution finds out that this technique gives the approximate value \( \sigma = 0.3296 \) with 0.73 accuracy. Asaithambi obtained this number correct to nine decimal positions as \( \sigma = 0.332057336 \). The variational iteration method (VIM) is implemented for a reliable treatment of two forms of Blasius equation. Fazio searched the Blasius problem numerically.

Main results

In this section, the solutions of (16)–(17) are presented in the \( W_2^4[0, \infty) \). On defining the linear operator \( L : W_2^4[0, \infty) \rightarrow W_2^4[0, 1] \) as

\[
Lv(t) = v^{(3)}(t) + \frac{\exp(-t) + t - 1}{2} v''(t) + \frac{\exp(-t)}{2} v(t)
\]

the problem (16) gets the form:

\[
\begin{cases}
Lv = f(t, u), & t \in [0, \infty), \\
v(0) = v'(0) = v'(\infty) = 0
\end{cases}
\]

where \( f(t, u) = \exp(-t) - \frac{1}{2} v(t) v''(t) - \frac{1}{2} \exp(-t)(\exp(-t) + t - 1) \). Let \( \psi_i(t) = T_i(t) \) and \( \psi'_i(t) = L^* \psi_i(t) \), where \( L^* \) is conjugate operator of \( L \). The orthonormal system \( \{\hat{\psi}_i(t)\}_{i=1}^{\infty} \) of \( W_2^4[0, \infty) \) can be obtained from Gram-Schmidt orthogonalization process of \( \{\psi_i(t)\}_{i=1}^{\infty} \).

\[
\hat{\psi}_i(t) = \sum_{k=1}^{i} \beta_{ik} \psi_k(t), \quad (\beta_{ii} > 0, \quad i = 1, 2, \ldots)
\]

Theorem 5. Let \( \{t_i\}_{i=1}^{\infty} \) be dense in \( [0, \infty) \) and \( \psi_i(t) = L y R_i(y) |_{y=t_i} \). The sequence \( \{\psi_i(t)\}_{i=1}^{\infty} \) is a complete system in \( W_2^4[0, \infty) \).

Theorem 6. If \( v(t) \) is the exact solution of (19), then

\[
v(t) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} f(t_k, v_k) \hat{\psi}_i(t).
\]

where \( \{(t_i)\}_{i=1}^{\infty} \) is dense in \( [0, \infty) \).

References


Some sequences of Fuzzy real numbers operated by a difference operator under an orlicz function

Bipul Sarma
Gauhati University, India
E-mail: drbsar@yahoo.co.in

Abstract In this article we introduce some I-convergent difference sequence spaces of fuzzy real numbers defined by Orlicz function and study their different properties like completeness, solidity, symmetricity etc.

References

Keywords: Fuzzy real numbers; I-convergence; solid space; symmetric space; Orlicz function.
2010 Mathematics Subject Classification: 40A05; 40D25; 46A45; 46E30.
Fuzzy Ideals in some specific $BE$–algebras

Kulajit Pathak

Barpeta Road Howly College, India

E-mail: kulajitpathak79@gmail.com

Abstract In this paper we introduced the concept of Cartesian product of $BE$–algebras and properties regarding commutativity, transitivity and self distributivity are obtained. We also study the concept of fuzzy ideals in Cartesian product of $BE$–algebras and the $BE$–algebra of all functions defined on an $99$algebra and find some new results.

References


Keywords: $BE$–algebra; cartesian product; ideal; Fuzzy ideal.

2010 Mathematics Subject Classification: 06F35; 03G25; 08A30; 03B5.
Some impulsive nonlocal control systems of Hilfer fractional orders

Amar Debbouche

Guelma University, Algeria

E-mail: amar_debbouche@yahoo.fr

Abstract  In this talk, we introduce a new concept called impulsive control inclusion condition, i.e., the impulsive condition is presented as inclusion related to multivalued maps and controls. The notion of approximate controllability of a class of semilinear Hilfer fractional differential control inclusions in Banach spaces is established. For the main results, we use fractional calculus, fixed point technique, semigroup theory and multivalued analysis. An appropriate set of sufficient conditions for the considered system to be approximately controllable is studied. Finally, we give an illustrated example to provide the obtained theory.

References


Keywords: Approximate controllability; Hilfer fractional differential inclusions; multivalued maps; semigroup theory; fixed-point; impulsive control inclusion conditions

2010 Mathematics Subject Classification: 93B05; 26A33; 34A60; 93C10.
Analytic propagation of epistemic uncertainty in queuing models

Baya Takhedmit¹, Sofiane Ouazine², Karim Abbas³

¹,²,³ Bejaia University, Algeria

E-mail: ¹bayatakh@gmail.com; ²wazinesofi@gmail.com; ³karabbas2003@yahoo.fr

Abstract This paper is devoted to the statistical study of the retrial $M/G/1/N$ queue. In other words, we use the Taylor series expansion, to estimate the stationary performances measures of the considered model, where some parameters of the queueing model are not assessed in a perfect manner. Specifically, we introduce the epistemic uncertainty in the service and the outgoing rates. Additionally, we approximate the expectation and the variance of the performance measures and we compare it to the corresponding Monte Carlo simulation results.

Introduction

The retrials queues are often employed to model many real situations such as computer systems, communication networks and so forth. These queueing models are generally solved at fixed parameters values. However, parameters values themselves are determined from a finite number of observations and hence have uncertainty associated with them (epistemic uncertainty). In order to effect an analysis of the propagation of the epistemic uncertainty of the parameters, through by calculating the stationary distribution in certain models of queues, we are considering to apply the analytical approach based on the Taylor series expansions of the Markov chains, and to validate the numerical results by the Monte-Carlo simulation technique. Specifically, we consider the $M/G/1/N$ retrial queue, where we assume, some parameters of the considered model are obtained under the epistemic uncertainty.

We introduce a new model that the parameters of the queuing system are represented by:

$$\theta = \bar{\theta} + \sigma_1 \varepsilon_1, \quad \mu = \bar{\mu} + \sigma_2 \varepsilon_2$$

where $\bar{\theta}$ and $\bar{\mu}$ are means of the parameters $\theta$ and $\mu$ respectively, $\sigma_1, \sigma_2$ are their standard deviations, $\varepsilon_1, \varepsilon_2$ are a random variables representing the white noises on computing the parameters $\theta, \mu$ ($\varepsilon_i \sim N(0, 1)$ for $i = 1, 2$).

Main results

We consider the $M/G/1/N$ retrial queue, where flow of arrivals is Poisson process with parameter $\lambda$, distribution of the service time is general distribution function $S$, with mean $\frac{1}{\mu}$, the duration between two successive recalls of the same secondary source with rate $\theta > 0$. The transition matrix is defined by [1]:

$$P(\theta, \mu) = \begin{pmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \cdots & p_{0,N-1} & 1 - \sum_{k=0}^{N-1} p_{0,k} \\ p_{1,0} & p_{1,1} & p_{1,2} & \cdots & p_{1,N-1} & 1 - \sum_{k=0}^{N-1} p_{1,k} \\ p_{2,0} & p_{2,1} & p_{2,2} & \cdots & p_{2,N-1} & 1 - \sum_{k=0}^{N-1} p_{2,k} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{N,0} & p_{N,1} & p_{N,2} & \cdots & p_{N,N-1} & 1 - \sum_{k=0}^{N-2} p_{N,k} \end{pmatrix}.$$
where $p_{i,j} = \frac{\lambda}{\lambda + \theta} k_{j-i} + \frac{\theta}{\lambda + \theta} k_{j-i+1}$, $k_n = \int_0^\infty \frac{(\lambda x)^n}{n!} e^{-\lambda x} dS(x)$, for $i = 0, 1, \ldots, j + 1$.

We consider the Taylor expansion of the stationary distribution $\pi$ in the neighborhood of the $(\bar{\theta}, \bar{\mu})$, which has become a random variable, given by:

$$
\pi(\theta, \mu) = \sum_{i=0}^n \sum_{j=0}^n \frac{1}{i! j!} \frac{\partial^{i+j} \pi}{\partial \theta^i \partial \mu^j} (\bar{\theta}, \bar{\mu}) (\sigma_1 \epsilon_1)^i (\sigma_2 \epsilon_2)^j + \sum_{i=n+1}^\infty \sum_{j=n+1}^\infty \frac{1}{i! j!} \frac{\partial^{i+j} \pi}{\partial \theta^i \partial \mu^j} (\bar{\theta}, \bar{\mu}) (\sigma_1 \epsilon_1)^i (\sigma_2 \epsilon_2)^j.
$$

(22)

In order to find a value of the order of development, we followed the steps described in the following algorithm:

**Algorithm 1. The remainder in the case of two variables**

**Begin**

**Inputs**: the precision $\zeta$, means $\bar{\theta}$, $\bar{\mu}$ and standard deviations $\sigma_1, \sigma_2$;

**Outputs**: the order $n$ of the Taylor polynomial, the value of the remainder $R_n(\theta, \mu)$;

1. Calculate the Taylor polynomial coefficients:

   $$
   \frac{\partial^{i+j} \pi}{\partial \theta^i \partial \mu^j} (\bar{\theta}, \bar{\mu}) = \sum_{k+l<n} C_k^n C_l^n \frac{\partial^{k+l} \pi}{\partial \theta^k \partial \mu^l} (\bar{\theta}, \bar{\mu}) D_{k+l}.
   $$

2. Calculate the remainder using the formula 22.

3. Calculate $\| E(R_n(\theta, \mu)) + Var(R_n(\theta, \mu)) \| > \zeta$ then

   Calculate the remainder using the formula 22.

**end while;**

**end.**

**References**


Some stochastic representations of quantum interference

Mustafa Moumni¹, Amrane Haoues²

¹,² University of Biskra, Algeria

E-mail: ¹m.moumni@yahoo.fr; ²haouesamrane@hotmail.com

Abstract  We study a class of differential equations which represents a quantum Yang-Mills system. We prove the term of interference (InterTerm in the equations) that makes the difference between the quantum case (considered here) and the classical one treated in previous works. We also demonstrate by schematizing this term of interference with a white noise, one can rewrite the equations in the form of a stochastic differential system. By using the Khasminski procedure, we show that the system admits solutions that no longer have chaotic behavior unlike the classical ones.

References


Keywords: Yang-Mills system; stochastic differential system; quantum interference.

2010 Mathematics Subject Classification: 82C10; 34K50.
Taylor series expansion approach for epistemic uncertainty propagation in queueing-inventory models

Massinissa Soufit¹, Karim Abbas²

Research Unit LaMOS, University of Bejaia, Algeria

E-mail: ¹massinissasoufit@gmail.com; ²kabbas.dz@gmail.com

Abstract  This paper discusses an approach based on Taylor series expansion for computing the uncertainty in performance measures of the M/M/1/N queueing models with inventory management, continuous review, exponentially distributed lead times and backordering, due to epistemic uncertainties in the service rate and the lead rate. Our proposal is to develop an algorithm allowing us to determinate lower moment corresponding to different performance measures of the considered model. Several numerical examples are carried out to illustrate the accuracy of the proposed approach. The obtaining numerical results are also compared to the corresponding Monte Carlo simulations results.

Introduction

Design and analysis of queueing-inventory models is often challenged by the fact that the exact values of the model parameters are either not known, or by lack of sufficient data for calibrating the model. In this paper we study the M/M/1/N queueing models with inventory management, continuous review, exponentially distributed lead times and backordering, where we assume that the service rate $\mu$ and the lead rate $\nu$ are not exactly known but data are available for estimating them. Hence, based on the data, we have an empirical distribution for the true values of these parameters. Under this assumption, to obtain a performance evaluation, we will use Taylor-series expansions for Markov chains. The Taylor-series coefficients are given in terms of the deviation matrix $[1]$.

Therefore, one is typically confronted with uncertainty about the true value of the parameters. This is known as "parameter uncertainty" in the literature, see, for example, $[2]$.

Main results

For $\theta \in \mathbb{R}^n$, let $P_\theta$ denote the transition matrix of a Markov model of the system under consideration. Denoting the stationary distribution of the discrete time Markov chain $P_\theta$ by $\pi_\theta$. The ergodic projector $\Pi_\theta$ is an transition kernel such that $\Pi_\theta P_\theta = \Pi_\theta$. The deviation matrix of a discrete time Markov chain $D_\theta$ is defined as

$$D_\theta = \sum_{n=0}^{\infty} (P_\theta^n - \Pi_\theta) = \sum_{n=0}^{\infty} (P_\theta - \Pi_\theta)^n - \Pi_\theta.$$  

(23)

Provided that the Markov chain has a single ergodic class and no transient states, $\Pi_\theta$ can be constructed by a matrix with row equal to $\pi_\theta$, the stationary distribution.

Keywords : queueing-inventory models; Markov chains; performance evaluation; perturbation analysis; Taylor-series expansions; epistemic uncertainty; Monte Carlo simulation.

2010 Mathematics Subject Classification : 60K25; 68M20; 90B05; 60J22.
We state the main result for finite Markov chains. Suppose the $P_0$ is element-wise differentiable and denote the matrix of mixed higher order derivatives is given by

$$\frac{\partial^n P_0}{\partial \theta_i \partial \theta_j^n}.$$  

**Theorem 7.** Let $\theta = (\theta_1, \theta_2) \in \Theta \subset \mathbb{R}^2$. Suppose that $P_0$ is element-wise $n$-times differentiable at $\theta_i$, $1 \leq i \leq 2$. The $n$th-order derivative of the stationary distribution $d^n \pi_0 / d \theta^n$, in terms of the deviation matrix $D_\theta$ of the associated discrete-time Markov chain on a finite discrete state space, is given as follows:

$$\frac{d^n \pi_0}{d \theta^n} = \sum_{k+j = n} (\frac{n-1}{k}) \pi^{(k+j)}(\theta) P^{(n-k-j)}(\theta) D_\theta,$$

where $P^{(k)}(\theta)$ is the matrix of the element-wise $k$th-order derivative of the transition matrix $P_0$ with respect to the parameter $\theta$.

In this paper, we discuss the multivariate Taylor series expansion for propagating the uncertainty in performance measures of the M/M/1/N queueing models with inventory management, continuous review, exponentially distributed lead times and backordering, due to epistemic uncertainties in the service rate $\mu$ and the lead rate $\nu$. More precisely, using the higher-order derivative of the stationary distribution with respect to multiple-parameter ($\mu, \nu$), which is introduced in [24].

We propose an approximate method based on Taylor series expansion for computing the expected value and the variance of the stationary distribution $\pi(\omega)$, which is function of random variables: $(\mu(\omega), \nu(\omega))$. In this sense, under the conditions of Theorem 7, the Taylor series expansion of the stationary distribution $\pi(\omega)$ of multiple-parameter $(\mu(\omega), \nu(\omega))$, can be written as follows:

$$\pi(\bar{\mu} + \sigma_\mu \epsilon_1, \bar{\nu} + \sigma_\nu \epsilon_2) = \pi(\bar{\mu}, \bar{\nu}) + \frac{\partial \pi}{\partial \mu} \sigma_\mu \epsilon_1 + \frac{\partial \pi}{\partial \nu} \sigma_\nu \epsilon_2 +$$

$$+ \frac{1}{2} \left[ \frac{\partial^2 \pi}{\partial \mu^2} \sigma_\mu^2 \epsilon_1^2 + \frac{\partial^2 \pi}{\partial \mu \partial \nu} \sigma_\mu \sigma_\nu \epsilon_1 \epsilon_2 + \frac{\partial^2 \pi}{\partial \nu^2} \sigma_\nu^2 \epsilon_2^2 \right]$$

$$+ \frac{1}{6} \left[ \frac{\partial^3 \pi}{\partial \mu^3} \sigma_\mu^3 \epsilon_1^3 + \frac{\partial^3 \pi}{\partial \mu^2 \partial \nu} \sigma_\mu^2 \sigma_\nu \epsilon_1^2 \epsilon_2 + \frac{\partial^3 \pi}{\partial \mu \partial \nu^2} \sigma_\mu \sigma_\nu^2 \epsilon_1 \epsilon_2^2 + \frac{\partial^3 \pi}{\partial \nu^3} \sigma_\nu^3 \epsilon_2^3 \right]$$

$$+ \ldots,$$

where

$$\mu(\omega) = \bar{\mu} + \sigma_\mu \epsilon_1(\omega), \nu(\omega) = \bar{\nu} + \sigma_\nu \epsilon_2(\omega),$$

with $\bar{\mu}$ (resp. $\bar{\nu}$) and $\sigma_\mu$ (resp. $\sigma_\nu$) represent the estimated mean value and the standard deviation associated with the random variable $\mu(\omega)$ (resp. $\nu(\omega)$), respectively, and $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ are a zero mean normal random variables, which modeling the epistemic uncertainty distribution.

**References**


An Encryption Algorithm for Gray-Scale Image Based On Bit-plane Decomposition and Diffuse Representation

Amrane Houas\textsuperscript{1}, Zouhir Mokhtari\textsuperscript{2}, Brahim Rezki\textsuperscript{3}

\textsuperscript{1,2,3} Laboratory of Applieds Mathematics PB 145 University of Biskra, Algeria

E-mail: \textsuperscript{1}haouesamrane@hotmail.com; \textsuperscript{2}zouhir_m@yahoo.fr; \textsuperscript{3}ibrahimrezki@yahoo.fr

Abstract In this short paper, we propose a new algorithm to encrypt gray-scale images. In the first step, we present a new basis to reduce the amount of data required to represent the image. In the second stage, we decompose the image into 8 bit-planes, and represent them in the new basis to obtain a key-image and encrypted images, the parameters obtained by this transformation are considered as key-image for the encryption and decryption algorithm. The decryption step is made by subtraction between each encrypted image and key-image to obtain the 8 bit-planes, with them we can restore exactly the original image without loss information. Experimental results introduced in this article demonstrate the effectiveness of the proposed scheme.

Introduction

Cryptography, or the art of encryption, is a science in itself. It consists in protecting messages, which can be understood by people who know the decryption method. This science is necessary for civilizations because there exist secrets. A secret sharing scheme (SSS) allows one to split a secret s into different pieces, called shares. Exchange of secret digital images are frequently used worldwide in a split second on the Internet. Therefore, it becomes very important to protect this information (image, printed text, handwritten notes, ...) against unauthorized users using cryptography. Cryptographic techniques can be divided into symmetric encryption (secret key) and asymmetric encryption (with private and public keys). Over the past few decades, information security has played an important role in social activities and has attracted more and more attention. As one of the hottest research topics, image encryption has been further developed by many scholars. In recent years, optical information processing technology has demonstrated its advantages in the field of image encryption due to its high processing speed and more degrees of freedom. Various optical techniques have been reported for image encryption. With the rapid development of the network multimedia, communication and propagation techniques, information security including image security is of serious concern. Image encryption is one of the effective approaches to ensure the security of the private image information. Visual cryptography is a secret sharing procedure for image data, which uses the properties of the human visual system to force the recognition of a secret message from overlapping encrypted images (shares). In existing schemes a well-known \((k, n)\)-threshold procedure is used to encrypt the secret image into \(n\) shares, which are then distributed amongst \(n\) recipients, any \((k−1)\) or fewer shares cannot be used to decrypt the transmitted information, visual sharing schemes cannot restore the transmitted information to its original quality when the original input is a natural image. This is due to the fact that the \((k, n)\)-threshold scheme operates on binary inputs and uses optical frosted/transparent representation. In this paper, we propose a new approach to encryption a gray-scale image, where we can restore the transmitted information to its original quality, that consists to decompose the original image into 8 bit-planes, represent them in the new basis proposed by Mokhtari and Melkemi. The matrix of parameters obtained by this transformation is considered as key-image and the images represented in this new basis are called encrypted images. For decryption, we do a subtraction between each encrypted image and the key-image, to obtain the 8 bit-planes, and we can restore exactly the original image.

Keywords: Cryptography; Image Encryption; Decryption; Security; Bit-plane Decomposition.

2010 Mathematics Subject Classification: 11T71; 68P25.
Main results

This section presents a class of methods to encrypt a gray-scale image. It is based on the transformation of the representation basis of given images; this transformation gives a representation that diffuses the images in the new basis.

In their work, they have shown that if we have a database \( \{I_k\}_{k=1}^d \) of \( d \) images represented in the orthonormal basis \( \{e_k\}_{k=1}^n \) such that for all \( k \):

\[
I_k = \sum_{j=1}^n a_{kj} e_j
\]

There exists a new base \( \{f_k\}_{k=1}^d \) where for all \( k \):

\[
I_k = \sum_{j=1}^n b_{kj} f_j,
\]

with \( b_{kj} = a_{kj} - a_{kj} \) and \( b_{kj} \sim \frac{\|I_k\|_1}{\sqrt{n}} \)

By choosing a suitable function and applying the method of least squares, they get the optimal settings \( j = 1, n \):

\[
a^* = \frac{1}{d} \sum_{k=1}^d \left( a_{kj} + \frac{\|I_k\|_1}{\sqrt{n}} \right)
\]

References


Mathematical modelling of methanol poisoning with impulsive dosing of antidote therapeutics to prevent toxicity

Priti Kumar Roy∗, Priyanka Ghosh, Shubhankar Saha

Centre for Mathematical Biology and Ecology, Department of Mathematics,
Jadavpur University, Kolkata - 700032, India

E-mail: *Corresponding Author: pritiju@gmail.com

Abstract Consumption of adulterated liquor leads to severe health risk causing increased morbidity and human causalities. Methanol is the major compound in liquor toxicity, causes severe metabolic disturbances, blindness and permanent neurologic dysfunction. It may be ingested due to addiction as an ethanol substitute by addicted people. It may be harmless but it is metabolized to formate in the presence of dehydrogenase enzymes which is greatly harmful. For affected addicts, toxicity of methanol is prevented by blocking the enzyme, alcohol dehydrogenase (ADH) through competitive inhibition approach along with other supportive care and restoring normalcy with metabolic clearance through urine. Based on enzymatic breakdown of methanol, we formulate a mathematical model with enzymatic competitive inhibition by ethanol as anecdote for the treatment of methanol toxicity. Initially, we study the non-negativity, boundedness, existence, local stability around the equilibrium point and global stability of the system for validation in reality. Impulsiveness through mathematical aspect has been applied for the conditions of permanency derived through enzyme inhibition model. Optimum time interval of ethanol dosing for detoxifying methanol poisoning has been determined from modified system. Finally, our results are confirmed by means of numerical simulation in accord with experimental findings.

Introduction

In this article, we firstly assume that alcohol dehydrogenase (ADH) binds with methanol as substrate. Ethanol which works as antidote in the body during methanol toxicity, prevents the reaction of enzyme and substrate. Thus, it is found that minimizing the methanol toxicity yielding toxic product. Now, considering the mechanistic reaction process, the enzyme kinetic model for the competitive inhibition is represented as follows:

\[
\begin{align*}
E + S & \xrightarrow{k_a} C \xrightarrow{k_b} E + P, \\
I & \xrightarrow{k_1} C_1 \xrightarrow{k_2} E + P_1,
\end{align*}
\]

where \(S, E, ES, P, I, EI\) and \(P_1\) are assumed as the methanol, enzyme, enzyme-methanol complex, formaldehyde, ethanol, enzyme-ethanol complex and acetaldehyde concentrations, respectively. We denote, the enzyme-methanol complex \((ES)\) concentration as \(C\) and enzyme-ethanol complex \((EI)\) concentration as \(C_1\) and \(k_a, k_{-a}, k_b, k_1, k_{-1}\) and \(k_2\) are positive rate constants for the enzyme kinetic reaction. In presence of ethanol, methanol binding affinity towards enzyme has been contested. Under the action of binding-affinity the reaction will follow ethanol controlled pathway. Hence, the competitive substrate reaction process will be monotonically inclined towards the inhibition...
pathway of the enzyme. Here, our main objective is to analyze how ethanol affects the system as an antidote with exogenous input. Different types of biologically feasible substrates are considered in literatures [1, 2], but here we consider a non-negative constant input as our dosing strategy. So considering the constant ethanol input ($I_c$), the interaction is represented by the following schematic diagram:

$$I_c \xrightarrow{k_1} C_1 \xrightarrow{k_2} E + P_1. \quad (27)$$

Therefore, the system of nonlinear ordinary differential equations for the above schematic diagram is represented by

$$\frac{dI}{dt} = I_c - k_1 EI + k_{-1}(E_0 - E),$$

$$\frac{dE}{dt} = -k_1 EI + (k_{-1} + k_2)(E_0 - E). \quad (28)$$

**Main results**

Important results that are mainly obtained in our analytical portion are stated as below:

**Theorem 8.** The solutions of system (28) with initial conditions $I(0) = I_0 > 0, E(0) = E_0 > 0, C_1(0) = 0$ and $P_1(0) = 0$ satisfy $I(t) > 0, E(t) > 0, C_1(t) > 0$ and $P_1(t) > 0$ for all $t > 0$. The region $R^2_E$ is positively invariant and attracting with respect to system (28).

**Theorem 9.** All positive solutions of system (28) with an initial value $(I_0, E_0)$ lie in the region $R^2_E$.

**Theorem 10.** The system (28) has an unique equilibrium point and no limit cycle if $R_E > 1$ in the plane $E - I$.

**Theorem 11.** The conditions of the permanence of impulsive induced system with initial conditions $I_0 > 0, E_0 > 0$, are $(k_{-1} + k_2)(E_0 - M) < k_1 M^2$ and $\frac{I_c}{\exp(k_1 E_0 \tau) - 1} < M$, where $M = M_0 + \frac{I_c}{\exp(\tau) - 1} = (2k_{-1} + k_2)E_0 + (I_0 + E_0) + \frac{I_c}{\exp(\tau) - 1} = (2k_{-1} + k_2 + 1)E_0 + I_0 + \frac{I_c}{\exp(\tau) - 1}$. □

Based on our analytical results, numerically we also focus on the safe dosage and dosing interval. To show the side-effects of wrong dosing inputs, we apply several dosage and dosing interval through impulsive way. We found that if we choose the ethanol dosing as enzyme inhibitor (maintenance dose of ethanol i.e. $I_c = 0.6 g/m/kg$) and dosing interval $\tau = 1 hr$, we can get pragmatic results to prevent methanol poisoning effect in the body.

**References**


On Some Difference and Discrete Equations and Related Boundary Value Problems

Vladimir Vasilyev
Belgorod National Research University, Russia
E-mail: vbv57@inbox.ru

Abstract  We introduce a discrete pseudo-differential operator in appropriate discrete functional spaces and study invertibility properties for such simplest operators in certain canonical domains of an Euclidean space. We construct special projectors for studying these operators in dependence on a type of canonical domain and show how these operators are related to special boundary value problems for holomorphic functions of several variables.

Introduction
Let \( D \subset \mathbb{R}^m \) be a sharp convex cone, \( D_d \equiv D \cap \mathbb{Z}^m \), and \( L_2(D_d) \) be a space of functions of discrete variable defined on \( D_d \), and \( A(\tilde{x}) \) be a given function of a discrete variable \( \tilde{x} \in \mathbb{Z}^m \). We consider the following types of operators
\[
(A_d u_d)(\tilde{x}) = \int_{T^m} \sum_{\tilde{y} \in D_d} e^{i(\tilde{y} - \tilde{x}) \cdot \xi} \tilde{A}(\xi) \tilde{u}_d(\xi) d\xi, \quad \tilde{x} \in D_d,
\]
and corresponding equation
\[
(A_s u_d)(\tilde{x}) = v_d(\tilde{x}), \quad x \in D_d,
\]
and introduce the function
\[
\tilde{A}_d(\xi) = \sum_{\tilde{x} \in \mathbb{Z}^m} e^{i\tilde{x} \cdot \xi} A(\tilde{x}), \quad \xi \in T^m.
\]
The function \( \tilde{A}_d(\xi) \) is called a symbol of the operator \( A_d \), and this symbol is called an elliptic symbol if \( \inf_{\xi \in T^m} |\tilde{A}_d(\xi)| > 0 \).
Our main goal is describing a periodic variant of wave factorization for an elliptic symbol \( \tilde{A}(\xi) \) and showing its usability for studying invertibility for the operator \( A_d \).

Main results
Let \( \tilde{D} \) be a conjugate cone, \( \tilde{D} = \{ x \in \mathbb{R}^m : (x, y) > 0, \forall y \in D \} \). The set \( T(D) \) is a subset of a multidimensional complex space \( \mathbb{C}^m \) such that \( T(D) = T^m + iD \).

Definition 1. Periodic wave factorization for elliptic symbol \( \tilde{A}(\xi) \) is called its representation in the form
\[
\tilde{A}_d(\xi) = \tilde{A}_d(\xi) \tilde{A}_e(\xi)
\]
where the factors \( A_{\pm}^{\pm}(\xi), A_{\pm}^{\pm}(\xi) \) admit bounded holomorphic continuation into domains \( T(\pm \tilde{D}) \).

Keywords: discrete pseudo-differential equation; symbol; factorization; boundary value problem.
2010 Mathematics Subject Classification: 35S15; 47G30.
Theorem 12. If the elliptic symbol \( \tilde{A}_d(\xi) \in C(\mathbb{T}^m) \) admits periodic wave factorization then the equation (29) has a unique solution in the space \( L_2(D_d) \) for arbitrary right-hand side \( v_d \in L_2(D_d) \).

Some preliminary studies for simplest operators and boundary value problems were done in previous author's papers [1,2].

Acknowledgments

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References


Methods in optimization, system theory & networks for mathematical modelling

Ioannis Dassios

University of Limerick, Ireland, University College Dublin, Ireland

E-mail: ioannis.dassios@ul.ie

Abstract In this talk, firstly I will present my latest results on areas that I currently work on such as optimization, fractional calculus, networks etc. Then we discuss how these results can be applied into mathematical models related to materials, image processing, power systems and macroeconomics.

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References


Keywords: optimization, fractional calculus, networks, differential equations.

2010 Mathematics Subject Classification: 26A33; 34A60; 34G25; 93B05.
Application of Two-Dimensional Wavelet Operational Matrices for Complex Partial Differential Equation

Vijay Kumar Patel¹, Vineet Kumar Singh²

¹,² Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi, India.

E-mail: ¹vijaybhuiit@gmail.com

Abstract
In this work, we described a complex partial differential equation (CPDE) with initial value conditions which is evaluated by continuous wavelet transform with Morlet wavelet. We develop an algorithm to obtain approximate numerical solution of CPDE with a numerical wavelet collocation method (NWCM) using a technique based on two-dimensional Legendre wavelet (TDLW) operational matrices. Firstly, we convert CPDE into system of partial differential equations (PDEs) and then, this system of PDEs converted into PIDEs. After that we reduced system of PIDEs into two algebraic equations via operational matrices. Finally, using NWCM, we find the solution of system of PDEs. Convergence of TDLW expansion are investigated and error analysis also. Illustrative examples are included to demonstrate the validity and applicability of the presented NWCM via operational matrices.

References


Keywords: CPDEs; Legendre wavelet; Collocation method; Operational matrix of differentiation and integration.

2010 Mathematics Subject Classification: 65N10; 45J05; 42C40.
Weak perturbation theory with applications to queuing

Badredine Issaadi¹, Karim Abbas², Djamil Aïssani³

¹,²,³ Bejaia University, Laboratory LAMOS, Targa Ouzemour, 06000 Bejaia, Algeria

E-mail: ¹issaadi_badredine@yahoo.fr; ²karabbas2003@yahoo.fr; ³lamos_bejaia@hotmail.com

Abstract  The calculation of the stationary distribution for a stochastic infinite matrix is generally difficult and do not have closed form solutions, it is desirable to have simple approximations converging rapidly to this distribution. In this paper, we use the weak perturbation theory to establish analytic error bounds in the GI/M/1 model and a tandem queue with blocking. Numerical examples are carried out to illustrate the quality of the obtained error bounds.

Introduction

Let $P = (P(i, j))_{i,j \geq 1}$, a stochastic infinite matrix, irreducible and positive recurrent, then it admits a unique stationary distribution $\pi = (\pi(j))_{j \geq 1}$, the calculation of this distribution is generally difficult if not impossible, it is desirable to have simple approximations converging rapidly to the distribution. For this, one solution is to approach $P$ by a finite stochastic matrix $P(k)$. Approximating an infinite Markov chain by using finite-state Markov chains is an interesting and often a challenging topic, which has attracted many researchers’ attention. Computationally, when we solve for the stationary distribution, when it exists, of an infinite-state Markov chain, the transition probability matrix of the Markov chain has to be truncated in some way into a finite matrix as a first step. We then compute the stationary distribution of this finite-state Markov chain as an approximation to that of the infinite-state one. The study of approximating the stationary probabilities of an infinite Markov chain by using finite Markov chains was initiated by Seneta [1] in 1967. Many up-to-date results were obtained by him and several collaborators. Other references may be found therein and/or in another paper [1] published in the same year by the same authors. Other researchers, including Wolf [3], used different approaches to those of Seneta et al. For instance, Heyman [4] provided a probabilistic treatment of the problem. Later, Grassmann and Heyman [2] justified the convergence for infinite-state Markov chains with repeating rows. All the above results are for approximating stationary distributions. Regarding more general issues of approximating a countable-state Markov chain, see the book by Freedman [7].

Numerical Example

In this work, we apply the weak stability method to problems of truncation [8]. We are interested in the study of the stability in the Markov chain constructed by augmenting the truncation in the last column of GI/M/1 system. We highlight the conditions for which it is possible to approach the characteristics of the ideal system by those corresponding of truncated and augmented model. After proving the fact of weak stability, we obtain the stability inequalities with an exact calculation of constants. Then, we have built algorithms to estimate the error due to the approximation. In order to examine the efficiency of the algorithms proposed, and to explore the stationary performance of the GI/M/1 model from performance of the truncated model, we provide a series of numerical results. For estimating the error on the stationary distribution of the GI/M/1 model by varying the size of the truncation ($k$). For
this, we considered four types of service time distributions, Deterministic (D), Exponential (M), Hyper-exponential (H2), Erlang (E2). Indeed, for the chosen distributions of service time, we obtain different coefficients of variation (CV).

As a second application we have chosen a tandem queue of an $M/M/1/\infty$ and $M/M/1/n$ queue. That is, at the first station an infinite number of customers is allowed but at the second no more than $n$. When the second station is saturated the servicing at the first station is stopped. Both stations have a single server and queueing is assumed to be first-in first-out. The service times at station 1 (resp. station 2) are exponentially distributed with parameters $\mu_1$ (resp. $\mu_2$). The interarrival times are also exponential but with a state dependent parameter $\lambda(i)$ when $i$ customers are present at station 1. Our objective is to evaluate the results that can be obtained after application of the weak stability method in the case of the tandem queue with blocking. This will allow us to judge the performance of this method. For this, we have developed two algorithms to estimate the perturbation bound obtained and to verify the conditions as well as the determination of their optimal stability domain.

Acknowledgments

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References


Integrability in terms of elementary functions of variable dissipation dynamical systems

Maxim V. Shamolin

Lomonosov Moscow State University, Russian Federation
E-mail: shamoin@rambler.ru

Abstract This activity is devoted to general aspects of the integrability of dynamical systems with variable dissipation. First, we propose a descriptive characteristic of such systems. The term “variable dissipation” refers to the possibility of alternation of its sign rather than to the value of the dissipation coefficient (therefore, it is more reasonable to use the term “sign-alternating”).

Introduction

We study nonconservative systems for which the usual methods of the study, e.g., Hamiltonian systems, are inapplicable. Thus, for such systems, we must “directly” integrate the main equation of dynamics. We generalize previously known cases and obtain new cases of the complete integrability in transcendental functions of the equation of dynamics of a four-dimensional rigid body in a nonconservative force field. Of course, in the general case, the construction of a theory of integration of nonconservative systems (even of low dimension) is a quite difficult task. In a number of cases, where the systems considered have additional symmetries, we succeed in finding first integrals through finite combinations of elementary functions. We obtain a series of complete integrable nonconservative dynamical systems with nontrivial symmetries. Moreover, in almost all cases, all first integrals are expressed through finite combinations of elementary functions; these first integrals are transcendental functions of their variables. In this case, the transcendence is understood in the sense of complex analysis, when the analytic continuation of a function into the complex plane has essentially singular points. This fact is caused by the existence of attracting and repelling limit sets in the system (for example, attracting and repelling focuses). We detect new integrable cases of the motion of a rigid body, including the classical problem of the motion of a multidimensional spherical pendulum in a flowing medium.

One of the definitions of a zero mean variable dissipation system

We study systems of ordinary differential equations that have a periodic phase coordinate. Such systems possess symmetries under which their average phase volume with respect to the periodic coordinate is preserved. For example, the following pendulum system, with a smooth and periodic (of period $T$) with respect to $\alpha$ right-hand side $V(\alpha, \omega)$ of the form $\ddot{\alpha} = -\omega + f(\alpha)$, $\dot{\omega} = g(\alpha)$, $f(\alpha + T) = f(\alpha)$, $g(\alpha + T) = g(\alpha)$, preserves its phase area on the phase cylinder over the period $T$: $\int_0^T \mathrm{div} V(\alpha, \omega) d\alpha = \int_0^T f'(\alpha) d\alpha = 0$. This system is equivalent to the equation of a pendulum $\ddot{\alpha} - f'(\alpha)\dot{\alpha} + g(\alpha) = 0$, in which the integral of the coefficient $f'(\alpha)$ of the dissipative term $\dot{\alpha}$ over the period is equal to zero. We see that this system has symmetries under which it becomes a system with variable dissipation with zero mean in the sense of the following definition.

Keywords: nonconservative dynamical system; dissipation; integrability; transcendental first integral.
2010 Mathematics Subject Classification: 34E; 70E.
Definition 2. Consider a smooth autonomous system of order \((n + 1)\) in the normal form defined on the cylinder \(\mathbb{R}^n \times S^1(\alpha \mod 2\pi)\), where \(\alpha\) is a periodic coordinate of period \(T > 0\). The divergence of the right-hand side \(V(x, \alpha)\) (which, in general, is a function of all phase variables and does not vanish identically) of this system is denoted by \(\text{div} V(x, \alpha)\). This system is called a system with variable dissipation with zero (respectively, nonzero) mean if the function \(\int_0^T \text{div} V(x, \alpha) d\alpha\) vanishes (respectively, does not vanish) identically. In some cases (for example, when at some points of the circle \(S^1(\alpha \mod 2\pi)\) singularities appear), this integral is meant in the sense of the principal value.

Main results

We define systems with variable dissipation with zero (nonzero) mean based on the divergence of the vector field of the system, which characterizes the change of the phase volume in the phase space of the system considered. We introduce a class of autonomous dynamical systems with one periodic phase coordinate possessing certain symmetries that are typical for pendulum-type systems. We show that this class of systems can be naturally embedded in the class of systems with variable dissipation with zero mean, i.e., on the average for the period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space, either energy pumping or dissipation can occur, but they balance to each other in a certain sense.

Then we study certain general conditions of the integrability in elementary functions for systems on the two-dimensional plane and the tangent bundles of a one-dimensional sphere (i.e., the two-dimensional cylinder) and a two-dimensional sphere (a four-dimensional manifold). Therefore, we propose an interesting example of a three-dimensional phase portrait of a pendulum-like system which describes the motion of a spherical pendulum in a flowing medium. For multi-parametric third-order systems, we present sufficient conditions of the existence of first integrals that are expressed through finite combinations of elementary functions \([1, 2]\).

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References


On the solution of fuzzy Volterra–Fredholm integral equation of second kind

Zahra Alijani¹, Urve Kangro²

¹University of Tartu, Estonia ²University of Tartu, Estonia

E-mail: ¹zahra.alijani2@gmail.com; ²urve.kangro@ut.ee

Abstract In this study we use orthogonal basis function set to solve second kind fuzzy integral equation that can be converted to a system of two in crisp case. We also consider collocation method for approximately solving the equation.

Introduction

The integral equations are used for variety of problems in physics, mechanics, economics, sociology, biology. These systems are dependent on a noise source, on a Gaussian white noise, so modeling such phenomena naturally requires the use of various Volterra integral equations. Most Volterra integral equations can not be solved analytically and hence it is of great importance to provide numerical solution. So, there has been a growing interest in numerical solutions of Volterra integral equations. Triangular functions approximation were successfully applied for analysis of dynamic systems, variational problems [1], integral equations [2], integro differential equations [3], Nonlinear Constrained Optimal Control Problem and Volterra-Fredholm integral equations.

The concept of integration of fuzzy functions was introduced by Dubois and Prade [2] and investigated by Goetschel and Voxman. We concentrate on using triangular basis function to solving linear fuzzy Volterra-Fredholm integral equations.

Main results

The Volterra integral equation of the second kind is

\[ y(t) = f(t) + \lambda \int_0^t K(s, t)y(s)ds, \quad t \in [0, T], \]

where \( \lambda > 0 \), \( K(s, t) \) is an arbitrary given kernel function over the triangle \( 0 \leq s \leq t \leq T \) and \( f(t) \) is a given function of \( t \in [0, T] \). If \( f(t) \) is a crisp function then the solution of above equation is crisp as well. If \( f(t) \) is a fuzzy function this equation may only posses fuzzy solution.

Let \( u(r) = (\underline{u}(r), \overline{u}(r)), \quad 0 \leq r \leq 1 \) be a fuzzy number, we take

\[ u^c(r) = \frac{\underline{u}(r) + \overline{u}(r)}{2}, \quad u^d(r) = \frac{\overline{u}(r) - \underline{u}(r)}{2}. \]

It is clear that \( u^d(r) \geq 0 \), also a fuzzy number \( u \in E \) is called symmetric if \( u^c(r) \) is independent of \( r \) for all \( 0 \leq r \leq 1 \). We obtain existence and numeric approximate solution for Eq. (30)

Keywords: Volterra-Fredholm integral equations, Triangular basis, collocation method

2010 Mathematics Subject Classification: 45D05; 45B05; 46S40.
By substituting Eq. (31) into (145), we have

\[
y^c(t, r) = f^c(t, r) + \lambda \int_0^t K(s, t) y^d(s, r) \, ds
\]

(32)

\[
y^d(t, r) = f^d(t, r) + \lambda \int_0^t |K(s, t)| y^d(s, r) \, ds
\]

(33)

Now we must solve two crisp Volterra integral equation of the second kind provided that each of Eqs. (32) and (33) have solution. We propose to use collocation method with triangular basis, where the collocation points are the equidistant points

\[ t_k = kh, \quad h = \frac{T}{n}, \quad k = 0, ..., N. \]

We look for solution of (30) in the form

\[ y^c_N(t, r) = \sum_{n=0}^{N} c_n(r) \phi_n(t) \]

and

\[ y^d_N(t, r) = \sum_{n=0}^{N} c'_n(r) \phi_n(t) \]

where \( \phi(t) \) is triangular basis function. Then the collocation equations are

\[
c_k(r) - \lambda \sum_{n=0}^{k} c_n(r) \int_0^{t_k} K(s, t_k) \phi_n(s) \, ds = f^c(t_k, r),
\]

(34)

\[
c'_k(r) - \lambda \sum_{n=0}^{k} c'_n(r) \int_0^{t_k} |K(s, t_k)| \phi_n(s) \, ds = f^d(t_k, r).
\]

(35)

Theorem 13. Let the kernel \( K : I \times I \to \mathbb{R} \) and the fuzzy function \( f : I \to E \) are continuous functions and \( \lambda \in \mathbb{R} \). Then Eq. (30) has a unique continuous fuzzy solution on \( I \).

Theorem 14. The collocation method with triangular basis converges for Volterra integral equations of the second kind with continuous kernels.

References


Finite element approximation to optimal pointwise control of parabolic problems with incomplete data

Sihem Mahoui\textsuperscript{1,2}, Mohamed Said Moulay\textsuperscript{1}, Abdennebi Omrane\textsuperscript{2}

\textsuperscript{1}USTHB, Algeria \quad \textsuperscript{2}Universit\textsuperscript{e} de Guyane, French Guiana

E-mail: mahoui.sihem@gmail.com; mmoulay@usthb.dz; abdenni.be.omrane@univ-guyane.fr

Abstract We study the a priori error estimates for finite element approximations of parabolic optimal pointwise control problems with incomplete data. The state equation exhibits low regularity due to the control imposed pointwisely, this introduces difficulties for both theoretical and numerical analysis. To solve the optimal control problem we use the low-regret control method of J-L. Lions. We discretize the optimal control problem by using piecewise linear and continuous finite elements for the space discretization of the state, and we use the backward Euler scheme for time discretization. We prove a priori error estimates for the low-regret pointwise control.

References


Keywords: Optimal control problem; low-regret control; pointwise control; finite element method; a priori error estimates.

2010 Mathematics Subject Classification: 35K05; 35K20; 49K20; 49K35; 93C20; 93C41; 65N30; 65N15.
Global tumor eradication conditions for cancer model under combined chemotherapy anti-angiogenic therapy

Konstantin E. Starkov

Instituto Politecnico Nacional, CITEDI, Mexico

E-mail: kstarkov@ipn.mx, konstarkov@hotmail.com

Abstract  Ultimate dynamics of the five-dimensional cancer tumor growth model at the angiogenesis phase is studied. This model describes interactions between normal cells, cancer cells, endothelial cells, chemotherapy agent and anti-angiogenic agent in tumor growth process. In this work we derive various ultimate bounds for cells populations and chemotherapy and anti-angiogenic concentrations. Global asymptotic tumor clearance conditions are obtained in two settings: in an application of chemotherapy and in a combined application of chemotherapy and anti-angiogenic therapy. Biological implications are discussed.

Introduction

In this paper we study the system elaborated by Pinho et al. in [1]

\[ \frac{dx_1}{dt} = a_1 x_1 (1 - x_1) - q_1 x_1 x_2 - p_1(x_3, w) \frac{x_1 y}{A_1 + x_1}, \]  

(36)

\[ \frac{dx_2}{dt} = a_2 x_2 (1 - \frac{x_2}{1 + \gamma x_3}) - q_2 x_1 x_2 - p_2(x_3, w) \frac{x_2 y}{A_2 + x_2}, \]  

(37)

\[ \frac{dx_3}{dt} = \beta x_2 + a_3 x_3 (1 - x_3) - \frac{p_3 x_3 w}{A_3 + x_3}, \]  

(38)

\[ \frac{dy}{dt} = \delta - [\xi + \frac{d_1 x_1}{A_1 + x_1} + \frac{d_2 x_2}{A_2 + x_2}] y, \]  

(39)

\[ \frac{dw}{dt} = \phi - [\eta + \frac{d_3 x_3}{A_3 + x_3}] w, \]  

(40)

with \( p_i(x_3, w) = p_{i0} + p_{i1} x_3 + p_{i2} w, i = 1, 2 \), which describes global dynamics of interactions between populations of normal cells \( (x_1) \), cancer cells \( (x_2) \), endothelial cells \( (x_3) \) at the angiogenesis phase under applying two types of treatment: by chemotherapy \( (y) \) and by anti-angiogenic \( (w) \) agents. All parameters are introduced in [1] as well. Our approach is based on the localization method of compact invariant sets, [2], the LaSalle theorem and M. Hirsch results concerning competitive systems. Main assertions concern ultimate dynamics of this system. In Theorem 1 we describe loci of compact invariant sets by a polytope. In Theorem 2 we establish tumor clearance conditions when only chemotherapy is utilized. In Theorem 3 we present global asymptotic tumor clearance conditions in the situation when both types of therapies are applied.

Keywords : cancer model; therapy; compact invariant set; global stability; competitive system.

2010 Mathematics Subject Classification : 92C50; 92C42; 34D23.
Main results

**Theorem 15.** Let $a_2 > \beta$. Then all compact invariant sets are located in the polytope in $\mathbb{R}^5_{+,0}$ defined by inequalities

$$x_1 \leq x_{1\text{max}} := 1; \quad x_2; x_3 \leq x_{2\text{max}} = x_{3\text{max}} := \frac{1}{a_2 - \beta} \left( \frac{(a_2 + a_3 + a_2\gamma - \beta)^2}{4a_3} + a_2 \right);$$

$$y_{\text{min}} := \frac{\delta}{\zeta + d_1 + d_2} \leq y \leq y_{\text{max}} := \frac{\delta}{\zeta}; \quad w_{\text{min}} := \frac{\phi}{\eta + d_3} \leq w \leq w_{\text{max}} := \frac{\phi}{\eta}.$$

**Theorem 16.** Suppose that $a_2 > \beta\gamma$ and treatment parameters $\delta$ and $\xi$ are positive and satisfy the following inequality

$$a_2 + (a_2 - \beta\gamma)A_2 + \frac{\gamma(a_2 + a_3)^2}{4a_3} < \sqrt{\frac{(a_2 - \beta\gamma)p_{20}\delta}{\zeta + d_1 + d_2}}.$$  

Then the $\omega$-limit set of each trajectory in $\mathbb{R}^5_{+,0} \cap \{w = 0\}$ is contained in the tumor-free plane $x_2 = 0$.

**Theorem 17.** Suppose that treatment parameters $\delta; \phi; \xi; \eta$ are positive and satisfy the following condition

$$p_{22}\delta\phi > (\eta + d_3)(a_2(A_2 + x_{2\text{max}})(\xi + d_1 + d_2) - p_{20}\delta) > 0.$$  

Then the $\omega$-limit set of each trajectory in $\mathbb{R}^5_{+,0} \cap \{y > 0; w > 0\}$ is contained in the tumor-free plane $x_2 = 0$.

Finally, we study the case when $\omega$-limit sets from Theorem 3 are equilibrium points and concern biological implications of our work.

Acknowledgments

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References


Fractional differential equations: Existence and uniqueness of solutions, with applications

Ricardo Almeida¹, Agnieszka B. Malinowska², M. Teresa T. Monteiro³

¹ University of Aveiro, Portugal  ² Białystok University of Technology, Poland  ³ University of Minho, Portugal

E-mail: ¹ricardo.almeida@ua.pt; ²abmalina@wp.pl; ³tm@dps.uminho.pt

Abstract  This talk deals with fractional differential equations, in terms of a fractional derivative with respect to another function. Existence and uniqueness results are proven. A Picard iterative method is exemplified and a Gronwall inequality is obtained. Then, some applications are exemplified. We develop some mathematical models to describe the world population growth and the gross domestic product of some countries, for such derivative.

Acknowledgments

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References


Keywords: Fractional differential equations, Gronwall inequality, modelling.

2010 Mathematics Subject Classification: 26A33, 34A08, 93E24.
Impulsive partial hyperbolic differential equations of fractional order with state dependent delay

Zohra Boutefal¹, Mouffak Benchohra²

¹Mascara University, Algeria  ²Sidi Bel Abbes University, Algeria

Abstract In this paper we provide sufficient conditions for the existence of solutions for partial functional differential equations with impulses and dependent delay, involving the Caputo fractional derivative. Our results will be obtained using suitable fixed point theorems.

Introduction

In the present article, we shall be concerned with the existence of solutions for the following impulsive partial hyperbolic differential equations:

\[( ^{c}D^{r}_{t_k} u)(t,x) = f(t,x,u(t,x,u(t,x),u_{t}(x),u_{x}(x))), \quad \text{if } (t,x) \in J_k, \ k = 0, \ldots, m, \]

\[u(t_{k}^{+},x) = u(t_{k}^{-},x) + I_{k}(u(t_{k}^{-},x)), \quad x \in [0,b], \ k = 1, \ldots, m, \]

\[u(t,x) = \phi(t,x); \quad \text{if } (t,x) \in \tilde{J} := [-\alpha, a] \times [-\beta, b] \setminus [0, a] \times [0, b], \]

\[u(t,0) = \psi(t), \ t \in [0, a], \ u(0,x) = \psi(x); \ x \in [0,b], \]

where \( J_0 = [0,t_1] \times [0,b], \) \( J_k := (t_k, t_{k+1}) \times [0,b], \) \( k = 1, \ldots, m, \) \( z_k = (t_k,0), \) \( k = 0, \ldots, m, \) \( J = [0,a] \times [0,b], \) \( \alpha, \beta > 0, \) \( 0 = \tau_0 < t_1 < \cdots < t_m < t_{m+1} = \alpha, \) \( ^{c}D^{r}_{t_k} \) is the Caputo fractional derivative of order \( r = (r_1,r_2) \in (0,1] \times (0,1], \)

\( \phi : [0,a] \rightarrow \mathbb{R}^n, \) \( \psi : [0,b] \rightarrow \mathbb{R}^n \) are given continuous functions with \( \phi(t) = \phi(t,0), \) \( \psi(x) = \phi(0,x) \) for each \( (t,x) \in J, \)

\( f : J 	imes C \rightarrow \mathbb{R}^n, \) \( \rho_1, \rho_2 : J \times C \rightarrow \mathbb{R}, \) \( I_k : \mathbb{R}^n \rightarrow \mathbb{R}^n, \) \( k = 1, \ldots, m, \) \( \phi : \tilde{J} \rightarrow \mathbb{R}^n, \) are given functions and \( C \) is the Banach space defined by

\[ C_{(\alpha,\beta)} = \left\{ u : [-\alpha,0] \times [-\beta,0] \rightarrow \mathbb{R}^n : \text{continuous and there exist } \tau_k \in (-\alpha,0) \text{ with } u(\tau_k,x) \text{ and } u(\tau_k,\tilde{x}), k = 1, \ldots, m, \text{ exist for any } \tilde{x} \in [-\beta,0] \text{ with } u(\tau_k,\tilde{x}) = u(\tau_k,\tilde{x}) \right\}. \]

Keywords: Impulsive partial differential equations, fractional order, solution, left-sided mixed Riemann-Liouville integral, Caputo fractional-order derivative, finite delay, infinite delay, fixed point.

2010 Mathematics Subject Classification: 26A33, 34K30, 34K37, 35R11.
Main results

We always assume that \( \rho_i : J \times C \rightarrow \mathbb{R} \), \( i = 1, 2 \) are continuous and the function \( (s, y) \mapsto u(s, y) \) is continuous from \( \mathcal{R} \) into \( C \).

Our first existence result for the IVP (41)-(44) is based upon the fixed point theorem due to Burton and Kirk.

Let us introduce the following hypotheses which are assumed after.

(H1) The functions \( I_k : \mathbb{R}^n \rightarrow \mathbb{R}^n \), and \( f : J \times C \rightarrow \mathbb{R}^n \) are continuous.

(H2) There exist \( p, q \in C(J, \mathbb{R}_+) \) such that

\[
\| f(t, x, u) \| \leq p(t, x) + q(t, x) \| u \|_C, \quad \text{for } (t, x) \in J \text{ and each } u \in C.
\]

(H3) There exists \( l > 0 \) such that

\[
\| I_k(u) - I_k(v) \| \leq l \| u - v \|_{PC} \quad \text{for each } u, v \in \mathbb{R}^n.
\]

Theorem 18. Assume that hypotheses (H1)-(H3) hold. If

\[
2ml < 1,
\]

then the IVP (41)-(44) has at least one solution on \([-\alpha, a] \times [-\beta, b]\).

Proof. We shall reduce the existence of solutions of (41)-(44) to a fixed point problem.

References


A mathematical model of tuberculosis improper treatment

Maman Fathurrohman

1 Universitas Sultan Ageng Tirtayasa, Indonesia

E-mail: mamanf@untirta.ac.id

Abstract  Tuberculosis is one of deadly infectious diseases. Treatment for this disease is using antibiotics (drugs) within duration of six months. Unfortunately, improper treatments of tuberculosis, for example inappropriate treatment by doctors or incomplete treatment by patients, will causing a worsening effect; namely Multi-Drug Resistant Tuberculosis (MDR-TB), in which the bacillus will resistant to the first line drugs, and the duration of treatment become longer. Furthermore, improper treatment of MDR-TB also will causing a similar worsening effect; namely Extensively-Drug Resistant Tuberculosis (XDR-TB), in which the bacillus will resistant to the second line drugs, the duration of treatment become longer than treatment duration of MDR-TB, and the disease will be more difficult to be cure. In this paper a model for worsening effects of improper treatment of tuberculosis is proposed and discussed.

References


Keywords: Tuberculosis; MDR-TB; XDR-TB.

2010 Mathematics Subject Classification: 37N25; 92B05.
On $p$-absolutely summable sequence space $\ell_p$ with statistical metric

Paritosh Chandra Das

Department of Mathematics, Rangia College, Rangia-781354, Assam, INdia

E-mail: pcdasrc2011@gmail.com

Abstract In this article we want to introduce the notion $p$-absolutely summable sequence space, $\ell_p$ with the concept of statistical metric and discuss some properties like completeness, solidness, symmetricity and convergence free.

Introduction

A real valued function $f$ on the set of real numbers is called a distribution function if it is non-decreasing, left continuous and has $\inf_{t \in \mathbb{R}} f(t) = 0$ and $\sup_{t \in \mathbb{R}} f(t) = 1$. Let $X$ be a non-empty set and $S$ denote the set of all distribution functions defined on $X$. Let $F$ be a mapping from $X \times X$ into $S$ and for every pair $(p, q)$ of $X$, we denote the distribution function $F(p, q)$ by $F_{pq}$ whence the symbol $F_{pq}(t)$ interprets as the probability that the distance from $p$ and $q$ is less than $t$. An ordered pair $(X, F)$ is called a statistical metric space (briefly, SM-space introduced by K. Menger) if it satisfies the following conditions:

1. $F_{pq}(t) = 1$ for all $t > 0$ if and only if $p = q$.
2. $F_{pq}(0) = 0$.
3. $F_{pq}(t) = F_{qp}(t)$.
4. $F_{pr}(t_1 + t_2) \geq T(F_{pq}(t_1), F_{qr}(t_2))$, for all $p, q, r$ in $X$ and $t_1, t_2 \geq 0$; where $T$ is a 2-place function on the unit square.

We want to introduce $p$-absolutely summable sequence, in statistical metric space as follows. A sequence $x = (x_k)$ in a statistical metric space $(X, F)$ is called a $p$-absolutely summable sequence if there exists $h > 0$ and $0 < \delta < 1$ such that $h > 1 - \delta$, where 0 (zero) is in $X$ and the distance $d$ between $x = (x_k)$ and 0 (zero) is defined by $d(x, 0) = \left\{ \sum_{k=1}^{\infty} |x_k|^p \right\}^{\frac{1}{p}}$

Main results

We discuss the following results here. The SM space, $(\ell_p, F)$ is

(1) Complete. (2) Solid and such is monotone. (3) Not convergence free. (4) Symmetric.

References


Keywords: Statistical metric; $t$-norm; solid space; symmetric space and convergence free.

2010 Mathematics Subject Classification: 40A05; 40A30; 60B10.
Dual combination phase synchronization of fractional order hyperchaotic systems with external disturbances

Vijay K. Yadav\textsuperscript{1} and Subir Das\textsuperscript{2}

\textsuperscript{1,2}Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India

E-mail: \textsuperscript{1}vijayky999@gmail.com; \textsuperscript{2}sdas.apm@iitbhu.ac.in

Abstract In this article the authors have studied the dual combination phase synchronization among fractional order hyperchaotic systems in the presence of external disturbances using nonlinear control method. The control functions are designed to achieve dual combination phase synchronization of fractional order hyperchaotic systems with the help of Lyapunov stability theory and a new lemma for Caputo derivative which is given for Lyapunov function of fractional order system. The numerical simulation and graphical results are carried out using MATLAB. The results exhibit the effectiveness and reliability of the method.

References


Keywords: Fractional derivative, Hyperchaotic system, Dual synchronization, Combination synchronization, Phase synchronization, Lyapunov stability theory.

2010 Mathematics Subject Classification: 26A33; 34C28; 34D06; 37B25.
Hybrid compound synchronization of fractional order complex chaotic systems

Subir Das\textsuperscript{1} and Vijay K. Yadav\textsuperscript{2}

\textsuperscript{1,2} Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India

E-mail: \textsuperscript{1}sdas.apm@iitbhu.ac.in; \textsuperscript{2}vijayky999@gmail.com

Abstract  The hybrid compound synchronization among fractional order complex chaotic systems has been studied. During synchronization the control functions are designed using active control technique. The chaotic attractors for complex systems are found for fractional-order time derivative, which is described in Caputo sense. The fractional order complex T-system, Lu system, Lorenz system and Chen system are taken to illustrate the hybrid compound synchronization. Numerical simulation results for different particular cases are depicted through graphs to demonstrate the effectiveness of the control technique during hybrid compound synchronization of fractional order complex chaotic systems.

References


Keywords: Fractional derivative, Complex chaotic system, Hybrid synchronization, Compound synchronization, Lyapunov stability theory

2010 Mathematics Subject Classification: 26A33; 34C28; 34D06; 37B25.
Stochastic approximation for equilibrium computation in a queue with strategic customers and delay-based reneging

Apostolos Burnetas
National and Kapodistrian University of Athens, Greece

E-mail: aburnetas@math.uoa.gr

Abstract We consider an unobservable markovian queue where arriving customers are strategic and decide whether to join or balk based on the reward for service, the expected cost of delay in the queue and the probability of reneging without obtaining service after they spend a predetermined amount of time in the queue. Under the reneging behavior the steady state behavior of the system, and the customer expected benefit as a function of the probability of entering the system is intractable. To solve the equilibrium equation, we develop an adaptive algorithm which combines simulation and stochastic approximation and converges almost surely to the unique symmetric equilibrium strategy.

Introduction

We consider the stochastic model of a service system with generally several servers, where customers arrive according to a Poisson process with rate $\lambda$ and service times are exponential with rate $\mu$. Arriving customers receive a reward $R$ for service completion and incur a cost $C$ per unit of time remaining in the system. Furthermore, after a customer spends time $\tau$ in the queue without having started service the service becomes useless and the customer abandons the system.

Arriving customers are strategic and decide whether to join or balk upon arrival. They do not observe the number of customers already present and maximize their net expected benefit.

Under the preceding assumptions the problem is to identify a symmetric join/balk customer strategy such that if all arriving customers follow it, then no customer has a strictly positive benefit by deviating from it. Such strategies are called symmetric equilibrium strategies (SES). In general a SES is a mixed strategy, i.e., arriving customers join the queue with probability $q$ and balk with probability $1-q$. The problem is to identify the equilibrium probability $q^e$. For a comprehensive review of customer equilibrium models in queueing systems see [1].

Under a mixed strategy $q$ the customers join the queue according to a Poisson process with rate $\lambda q$. However, because the reneging time of each customer depends on the arrival time of the specific customer (i.e., $\tau$ time units after arrival if service has not started), the state information must include the arrival times of the customers waiting in the queue.

Let $P_q$ and $E_q$ denote probability and expectation in steady state under a mixed strategy $q$. Furthermore, let $T_0$, and $T$ denote the waiting time in the queue and the sojourn time in the system of a customer. Then the expected benefit of a tagged customer who uses strategy $q'$ when all other customers use strategy $q$ is equal to $B(q';q) = q'H(q)$, where

$$H(q) = RP_q(T_0 \leq \tau) - E_q(T).$$

A strategy $q^e$ is a SES if it maximizes the expected benefit of any arriving customer assuming that all other customers follow it, i.e., if

**Keywords**: queueing theory; strategic behavior; reneging customers; stochastic approximation

**2010 Mathematics Subject Classification**: 60K25; 90B22; 62L20.
\[ B(q^e; q^e) = \max_{q' \in [0,1]} B(q'; q^e). \]

Main results

We first show that \( H(q) \) is decreasing in \( q \). Based on this property, we show that there exists a unique SES \( q^e \), which is determined as follows:

1. If \( H(0) \leq 0 \), then \( q^e = 0 \).
2. If \( H(q) \geq 0 \), then \( q^e = 1 \).
3. If \( H(1) < 0 < H(0) \), then \( q^e \) is the unique solution of \( H(q^e) = 0 \).

Because of the complicated state description, the computation of \( H(q) \) is intractable both analytically and numerically, rendering the computation of the equilibrium strategy very difficult. We propose a variant of the stochastic approximation method, introduced by Robbins and Monro \(^2\), which is based on adaptive selection of \( q \) together with simulation in order to solve the equilibrium equation. In particular a simulation model of the system is constructed, where each arriving customer selects a join probability which is a perturbation of that of the previous customer, based on the observed delays and reneging behavior. It is shown that under appropriate selection of the perturbation mechanism, the sequence of join probabilities converges almost surely to the SES \( q^e \).

References


The role of HPV on cervical cancer with several functional response: a comparative study

Sudip Chakraborty, Priti Kumar Roy

Department of Mathematics,
Jadavpur University, Kolkata - 700032, India.

E-mail: sudip2205@gmail.com

Abstract Cervical cancer is one of the common cancers in female now a days. The development of cervical cancer cells from normal cells is caused by Human Papilloma Virus (HPV) and the progression can be described using mathematical model at a cellular level. We develop a mathematical model consisting of five compartments to describe the interactions between Human Papilloma Virus and four classes of epithelial and basal cells (susceptible, infected, precancerous and cancerous) of cervix. In this system, we consider that the disease transmission rate from precancerous to cancerous cells is governed by different types of functional response according to the risk and our cell immunity power which is dependent on the antibody genes p-53 and p-Rb. So we have consider three type of functions such as linear, Holling type II and Holling type III. We analyze the local stability of the equilibrium points of each of the types in a comparative way and investigate numerically the parameters which play an important role in the progression towards the cervical cancer. Furthermore, the method of perturbation, based on the existence of different time scale, is used to understand how a small change in our system by external interruption can effect the cervical cancer progression.

Introduction

Human Papilloma-virus (HPV) is a family of virus that includes more than 170 different types of virus and among them 40 types of virus are sexually transmitted. Genital HPVs, which are transmitted sexually, are the primary factor in cervical cancer worldwide. Cervical cancer is a cancer arising from the cervix. It is due to the abnormal growth of cells that have the ability to invade or spread to other parts of the body. In this research work, we consider a HPV infection and cervical cancer development model in cervix taking susceptible cells(S), infected cells(I), HPV virus(V), precancerous cells(P) and cancerous cell(C). We assume that the susceptible cell population takes a logistic growth with growth rate r and carrying capacity K. HPV infects the susceptible cells at the rate of λ and HPV infection can autoimmune at the rate of ρ. We assume that the infected cells becomes precancerous at a progression rate β and from precancerous to cancerous cells at a maximal rate θ. α1, δ1; α2, δ2 and α3, δ3 are the proliferation and apoptosis rates of infected, precancerous and cancerous cells respectively. Let the birth rate of virus be η and d be the death rate of HPV.

Thus we have the following mathematical model:

Keywords: Human Papilloma Virus (HPV), cervical cancer, basic reproduction ratio, Holling type functional response, Perturbation.

2010 Mathematics Subject Classification: 14E05; 34D10.
\[ \frac{dS}{dt} = rS(1 - \frac{S + I}{K}) - \lambda SV + \rho I, \]
\[ \frac{dI}{dt} = \lambda SV + (\alpha_1 - \delta_1)I - \beta I - \rho I, \]
\[ \frac{dV}{dt} = \eta(\alpha_1 - \delta_1)I - dV, \]
\[ \frac{dP}{dt} = \beta I + (\alpha_2 - \delta_2)P - \theta f(P), \]
\[ \frac{dC}{dt} = \theta f(P) + (\alpha_3 - \delta_3)C. \]  

(46)

Where \( S(0) > 0, I(0) \geq 0, V(0) \geq 0, P(0) \geq 0, C(0) \geq 0 \) and all the parameters are assumed to be non-negative. We consider the function \( f(P) \) is of three different following types: Linear, Holling type II and Holling type III.

**Main results**

Analysis of the model has shown that the model can exhibit many different behaviors, depending on the basic reproductive number of the infection \( R_0 \), the proliferation rate of precancerous cell \( \alpha_2 \) and the maximal progression rate \( \theta \) of precancerous cells to cancerous cell. Our numerical studies have focussed that HPV infections can generate a high risk of cervical cancer. Numerically we show the trend of the infected and cancerous cell population for different values of \( \phi \) for the Holling type-II functional response and how the parameter \( \phi \) plays a crucial role on both infected and cancerous cell population. Our result shows that as \( \theta \), the progression rate from pre-cancerous to cancerous cell population, increases the cancerous cells increases rapidly from same range of time. Also after the comparison, we observe that the growth of cancerous cell is low for Holling type II and III functional response compared to the linear type functional response.

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**References**


Kernel basis functions method for coupled mathematical models of heat and mass transfers

Vladimir I. Gorbachenko¹, Oleg E. Iaremko², Mohie M. Alqezweeni³

Penza State University, Russia

E-mail: ¹gorvi@mail.ru; ²yaremki@mail.ru; mohieit@mail.ru

Abstract  We get the solution of the Cauchy problem in the form of a weighted sum for fundamental solutions generated by single sources localized in any points at time. The error of the Cauchy problem solution is the same as the error for the initial conditions in the uniform metric. In the article, method of matrix kernel basis functions is developed. Using this method, we solve vector Cauchy problem for the heat equation on the real axis with the division points for a multilayer media. The initial conditions are approximated by weighted sum of kernel fundamental matrices satisfied matching conditions.

Introduction

Interpolation basis function plays an essential role in most fields of computational sciences and engineering. Interpolation basis function data is an essential component in processing and numerical solution of partial differential equations. The traditional basis functions are mostly coordinate functions, such as polynomial and trigonometric functions. Instead, radial basis functions (RBF) [1] are constructed in terms of a one-dimension distance variable. RBF method appears to have a clear edge over the traditional polynomial basis functions. RBF approximation functions method for multidimensional case has advantages over the traditional method. Kernel basis functions method [2] is a special case of RBF approximation functions method. We consider the kernel basis functions, which are generated by the fundamental solution [3] of the heat equation. A fundamental solution, also called a heat kernel, is a solution of the heat equation corresponding to the initial condition of an initial point source of heat at a known position. These can be used to find a general solution of the heat equation. One can obtain the general solution of Cauchy problem for one variable heat equation by applying a convolution.

Main results

Kernel basis functions are obtained by quadrature of convolution. Quadrature formula node is the Gaussian center, a time variable is interpreted as the Gaussian window width. The heat equation is invariant with respect to shifts in time variable. Then the solution of the Cauchy problem will be approximated by a weighted sum of the fundamental solutions generated by single sources localized at different points at different times. In this case, the approximation of the Cauchy problem solution is the heat equation solution, and there arises the problem of the initial condition approximation by Gaussians’ weighted sums. If you find an approximation of the initial condition with a given error in the uniform metric, then with the same error of Cauchy problem solution will be obtained. In this paper, we extend the kernel basis functions method into the case of the Cauchy problem for the systems of heat equations on the real axis. We use an approach in which the fundamental solution is replaced by fundamental matrix. Vector function approximation problem appears by using a weighted sum of the fundamental kernel matrices.

Keywords: fundamental solution, fundamental matrix, kernel basis functions, Gaussian, approximation.

2010 Mathematics Subject Classification: 65L05;65M70, 65M80.
A weighted sum is understood as the sum of product for the fundamental kernel matrices and the unknown weight vector coefficients. Setting weight vector coefficients, center and window width of each Gaussian carried out by least square method (LS method). In this paper we consider one more way of approximating the vector function. This method consists in the fact that we replace the vector Cauchy problem into n disjoint Cauchy problems, then we find an approximation of each scalar problem and the solution of a vector problem. Vector Cauchy problem for the heat equation on the real line with n division points to a multilayer medium is solved. Here, the fundamental matrix calculation causes difficulties. We developed the modification of kernel matrix functions method. Initial conditions are approximated by a weighted sum of the fundamental kernel matrices satisfying matching conditions. The fundamental matrix on the right infinite interval is chosen to be the fundamental matrix for single occasion. The fundamental matrix is uniquely determined by matching conditions on the remaining intervals.

References


A non-standard finite difference method for digital put option pricing under the fractional black-scholes model

Sedaghat Shahmorad 1, Robab Kalantari 2

1 University of Tabriz, Iran 2 University of Tabriz, Iran

E-mail: 1shahmorad@tabrizu.ac.ir; 2 r_kalantari@tabrizu.ac.ir

Abstract We introduce the mathematical model of digital put option pricing under the Fractional Black-Scholes (FBS) model. The digital put option pricing is option pricing where the payoff is non-smooth, the optimal exercise boundary is equal with exercise price. Following this, we introduce the non-standard Grünwald Letnikov approximation and use this approximation for time fraction term in FBS model to reach a fractional non-standard finite difference (NSFD) problem. We show that the proposed fractional NSFD scheme is stable and convergent. Uniqueness of the approximate solution is also proved. We also show that numerical results satisfy the physical conditions of digital put option pricing under the FBS model.

Introduction

The digital put option pricing under the FBS model are presented as

\[
\frac{\partial^\alpha P}{\partial t^\alpha} = \left( rP - rS \frac{\partial P}{\partial S} \right) t^{1-\alpha} \frac{\alpha!}{(1-\alpha)!} - \frac{\alpha^2}{2} \sigma^2 S^2 \frac{\partial^2 P}{\partial S^2}, \quad S > E, \quad 0 \leq t < T, \tag{47}
\]

\[
P(S, T) = \begin{cases} 
1 & S \leq E, \\
0 & S > E.
\end{cases} \quad P(E, t) = 1, \quad \lim_{S \to \infty} P(S, t) = 0, \tag{48}
\]

\[
P(S, t) = 1, \quad 0 \leq S \leq E, \tag{49}
\]

where \(P(S, t)\) is the digital put option pricing. The parameters \(\sigma, r\) and \(E\) show the volatility of the underlying asset, the interest rate and the exercise price of the digital option, respectively. The details of obtaining the FBS model are explained in [1].

Main results

We consider [17, 19] and use a NSFD scheme for derivatives on the right side and a non-standard Grünwald Letnikov approximation for fractional derivative on the left side of (47). To do this let \(t_k = k\Delta t, k = 0, 1, 2, \ldots, n-1\) and \(S_j = j\Delta S, j = 1, 2, \ldots, m-1\), where for \(0 \leq t \leq T, \Delta t = \frac{T}{n}\), and \(\Delta S = \frac{S_{\text{max}}}{m}\) are time and stock price steps respectively. Then instead of step sizes \(\Delta t\) and \(\Delta S\), we use the functions

\[
\phi(\Delta t) = 1 - e^{-\Delta t}, \quad \psi(\Delta S) = e^{\Delta S} - 1, \quad \phi(\Delta S^2) = 4\sin^2\left(\frac{\Delta S}{2}\right), \tag{50}
\]

Keywords: Fractional Differential Equation; Digital Option Pricing; Non-Standard Finite Difference Method; Interpolation Method.

2010 Mathematics Subject Classification: 26A33; 62P05; 34G25; 34A60.
to get from \((47)\)

\[
(1 - e^{-\Delta t})^{-\alpha} \sum_{i=0}^{k+1} b_i^\alpha P_j^{k+1-i} = \left( r P_j^k - r S_j \right) \frac{p_j^k - p_{j-1}^k}{(e^{\Delta S} - 1)} \left( \frac{1}{1 - \alpha} \right) + \alpha \frac{1}{2} \sigma^2 S_j \Delta S_{j+1} \frac{p_j^k - 2p_j^k + p_{j+1}^k}{(2 \sin(\frac{\Delta S}{2}))^2}, \quad S_j > E, \quad 0 \leq t_k < T,
\]

subject to

\[
P(S_j, T) = \begin{cases} 1 & S_j \leq E, \\ 0 & S_j > E. \end{cases}, \quad P(E, t_k) = 1, \quad \lim_{S \to \infty} P(S_j, t_k) = 0, \quad 0 \leq S \leq E.
\]

If \(j = 0\), then for all time values, it also follows from \((49)\) that \(P_0 = 1\) (since \(S_0 = 0\)) and if \(S_1 = \Delta S \leq E\) then from Eq. \((51)\) we have \(P(S_1, t) = 1\) and so

\[
CP_{j+1} = AP_j + BP_{j-1}, \quad P_0 = 1, \quad P_1 = 1.
\]

**Theorem 19. (Stability)** If \(\Delta S \leq E\) then the NSFD scheme \((54)\) is stable and we have \(\|E_{j+1}\|_{\infty} \leq K \max\{\|E_0\|_{\infty}, \|E_1\|_{\infty}\}, j = 2, 3, \ldots, m - 1\), where \(K\) is a positive constant independent of \(\Delta t, \Delta S\) and \(j\).

**Theorem 20. (Convergence)** Let \(P_j^k\) have the smooth solution \(P(S, t) \in C^{2, \alpha}_{S, t}(\Omega)\). Let \(P_j^k\) be the numerical solution computed by use of \((54)\). Then \(P_j^k\) converges to \(P(S_j, t_k)\), if \(\Delta S \leq E\).

We can obtain

\[
P_k = (I - DA)^{-1} \left( - \sum_{i=1}^{k-1} A_i^{(\alpha)}(P^{k-i}) + P_0^k \sum_{i=0}^{k-1} A_i^{(\alpha)} + P_0 \right).
\]

**Theorem 21. (Uniqueness)** If the matrix \(I - DA\) is invertible, then Eq. \((55)\) has a unique approximate solution.

To solve \((51)-(53)\), we use a higher order Grünwald Letnikov approximation. Since the digital put option pricing is only known at the end point (exercise time), then in order to use a higher order Grünwald Letnikov approximation, we need some other points of digital put option pricing. To get these intermediate values, we use a suitable interpolation method, that is, for the points \(P_0 = P(S, 0)\) and \(P_T = P(S, T)\).

**References**

Dynamic stress intensity factor due to applied point loading in a cracked infinite orthotropic elastic strip

Prashant k. Mishra¹ and Subir Das²

¹,² Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India

E-mail: ¹prshntmshr58@gmail.com; ²sdas.apm@iitbhu.ac.in

Abstract  This article deals with the investigation of elasto-dynamic response of a finite crack embedded in an infinite orthotropic strip under suddenly applied stress. The crack is situated symmetrically and oriented in a direction normal to the edges of the strip. Integral transforms are employed to reduce the transient problem to a pair of dual integral equations in the Laplace transformed plane which are solved by iterations in the low frequency domain. To determine time dependence of the parameters, these equations are inverted to yield the analytical expression of the dynamic stress intensity factor and crack opening displacement (COD). These physical quantities are calculated for different point loading given on the surface of the crack for the composite materials graphite epoxy and glass epoxy. The numerical values thus obtained are depicted through graphs for different particular cases.

References


Keywords: Orthotropic elastic strip; Impact response; Dynamic stress intensity factor.

2010 Mathematics Subject Classification: 74Bxx; 74Rxx; 74Sxx.
Solving the equation $f(x) = \alpha$ in a functional space using an iterative algorithm

Kheira Mecheri$^1$, Ahmed Ait Saidi $^2$

$^1$University of Bejaia, Algeria $^2$University of Bejaia, Algeria

E-mail: $^1$berkailili@yahoo.fr; $^2$haitsaidi@yahoo.fr

Abstract In this note, we establish exponential inequalities of the Bernstein-Frechet type for the procedure of Robbins-Monro, through of a single-functional index, when this procedure is valued in a separable Hilbert space. These inequalities make it possible to show the almost complete convergence (with rate).

Introduction

through the functional index $\theta_0$, to resolve $f(X) = \alpha$ when $X$ is valued in an infinite-dimensional space. But, instead of considering a pure nonparametric equation, we prefer to introduce the following single-functional index equation

$$f(\langle \theta_0, X \rangle) = \alpha$$

where $\langle . , . \rangle$ is the inner product, $f$ is a real-valued function, $\theta_0$ is the single-functional index fixed in $\mathcal{H}$ and $X$ is a functional random variable valued in a $\mathbb{R}$-separable Hilbert space $\mathcal{H}$. Such a single-functional index equation assumes that the solution $x_0$ will be approximated

Algorithm and asymptotic study

To estimate the $x_0$ root of the equation (1), we introduce a stochastic algorithm of Robbins-Monro similar to that introduced in [3] and defined by

$$\langle \theta_0, X_{n+1} \rangle = \langle \theta_0, X_n \rangle - \frac{a}{n} (Y_{n+1} - \alpha)$$

where

$$Y_{n+1} = f(\langle \theta_0, X_n \rangle) + \xi_{n+1}.$$ 

and the real sequence $(\xi_n)_{n \in \mathbb{N}}$ is independent and identically distributed with zero mean and $a$ is a positive constant. With loss of generality, let us suppose $\alpha = 0$.

Let us introduce now the following assumptions :

(H1) The parameter $x_0$ checks a priory

$$|\langle \theta_0, X_1 - x_0 \rangle| \leq N < +\infty.$$ 

(H2) $f$ is a function satisfying

$$\forall x \in \mathcal{H}, \quad 0 < m \leq \frac{f(\langle \theta_0, x \rangle)}{\langle \theta_0, x - x_0 \rangle} \leq M < +\infty.$$ 

Keywords: Kernel estimator, single-functional index model, exponential inequalities of the Bernstein-Frechet.

2010 Mathematics Subject Classification: 26A33; 34A60; 34G25; 93B05.
(H3) We suppose that, for any $\epsilon > 0$,
\[ \phi_n(\epsilon) = n^{am} e^{a\gamma \epsilon} - N > 0 \]  
(61)
where $\gamma$ is the Euler constant.

(H4) The sequence $(\xi_n)_{n \in \mathbb{N}}$ is independent and identically distributed with zero mean and satisfying
\[ E(|\xi_1|^2) = \sigma^2 < +\infty \]
and the following Cramer condition
\[ E(|\xi_1|^p) \leq \frac{p! \sigma H^{p-2}}{2} < +\infty, \quad p \geq 2, \quad p \in \mathbb{N} \]
with $H$ is a positive constant and $E$ is the mean operator. Not that assumption (H4) is fulfilled when $(\xi_n)_{n \in \mathbb{N}}$ is bounded or is a Gaussian process. We can know state the following results:

**Theorem.** Under the assumptions (H1)-(H4), we have
\[ P[|\langle \theta_0, X_{n+1} - x_0 \rangle| > \epsilon] \leq C_1 \exp\left(-C_2 n^{am} \epsilon\right) \]  
(62)
where
\[ C_1 \text{ and } C_2 \text{ denote some strictly positive constants.} \]

**Corollary 1.** Under the assumptions of the theorem we have
\[ \lim_{n \to +\infty} \langle \theta_0, X_{n+1} \rangle = \langle \theta_0, x_0 \rangle, \ a.c.o. \]  
(63)

**Corollary 2.** Under the assumptions of the theorem we have
\[ |\langle \theta_0, X_{n+1} - x_0 \rangle| = O_{a.c.o.} \left( \frac{\log n}{n^{am}} \right). \]  
(64)

**References**


The performance measures of some queues with preemptive priority

Naima Hamadouche, Djamil Aissani

1 University of Bejaia, Algeria  2 University of Bejaia, Algeria

E-mail: 1naima_maths@yahoo.fr; 2djamil.aissani@univ-bejaia.dz

Abstract In this work, we consider the M , M / M , M / 1 queue with preemptive priority, firstly we calculate the stationary distribution of the system, in a second step, we use this distribution to calculate performance measures of the system, such as waiting times for priority and non-priority customers.

Introduction

In several computer and telecommunication systems customers can be grouped to classes. Customers belonging to different classes can have different behavior, e.g. different arrival process, or different service requirement. In server stations the server can take the class of the customers into considerations when deciding the service order. For example, a given customer class can represent urgent customer, and the server can serve them before customers belonging to other classes.

There are a great number of papers and books which study priority queueing systems. The initials works are [2]. In all of these works the arrival process is assumed to be a Poisson process. Recently several papers have been devoted to the investigation of this kind of models (see for example [1] and references therein). In this work, we consider the M , M / M , M / 1 queue with preemptive priority, firstly we calculate the stationary distribution of the system, in a second step, we use this distribution to calculate performance measures of the system, such as waiting times for priority and non-priority customers.

Main results

Consider an M2/M2/1 queue with preemptive priority, in which there are two classes of customers high and low priority who arrive under independent Poisson processes with parameters, respectively, \( \lambda_1 \) and \( \lambda_2 \). We assume that, no low-priority customer enters service when any high-priority customers are present. If a low-priority customer is in service, his service will be interrupted at once if a high-priority customer arrives. Service times of the both kinds of requests are independent, and follows an exponential distribution with parameter \( \mu_1 \), respectively \( \mu_2 \), and with probability densities \( E_{\mu_1}(t) \) and \( E_{\mu_2}(t) \) respectively.

Our analysis focuses upon the number of requests just at the instant \( t_{n+1} \), service completions of "priority (respectively non priority) requests" or "instant of interruption". The state of this queue is describing by a semi-Markov process, which is an imbedded Markov chain (the number of requests present in the system) at the instants \( t_{n+1} \).

For that we consider:

To compute the probability of transition, \( P_{kj}(i,j) \), we have the following cases:

Keywords: Preemptive queue, Markov chain, stationary distribution, performance measures.

2010 Mathematics Subject Classification: 60K25; 60J20; 68M20.
1. $X_n^1 = k > 0, X_n^2 = 1 > 0$:

The transition probability $P_{k,i}(i, j)$ is given by:

$$P_{k,i}(i, j) = P(X_{n+1}^1 = i, X_{n+1}^2 = j | X_n^1 = k, X_n^2 = l),$$

$$= \int \frac{(\lambda_1 t)^{i-k+1} (\lambda_2 t)^{j-l}}{(i-k+1)! (j-l)!} e^{-(\lambda_1 + \lambda_2)t} E_{\mu_1}(t) dt.$$ (65)

2. $X_n^1 = 0, X_n^2 = 1 > 0$:

The transition probability $P_{k,i}(i, j)$ is given by:

$$P_{(0,l)}(0, j) = \int e^{-\lambda_1 t} (\lambda_2 t)^{j-l+1} (j-l)! e^{-\lambda_2 t} E_{\mu_2}(t) dt, \ j \geq l - 1,$$

$$P_{(0,l)}(1, j) = \int \int \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-(\lambda_2 t)} e^{-\lambda_1 t} t E_{\mu_2}(t) dt dt, \ j \geq l,$$ (66)

$$P_{(0,l)}(i, j) = 0, i \geq 2.$$

3. $X_n^1 = 0, X_n^2 = 0$:

Denote by "A" the event "first arrival is a priority request", then $P(A) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. For which, we have:

$$X_{n+1}^1 = A_{n+1}^1, \ X_{n+1}^2 = A_{n+1}^2.$$

On the other hand, if we denote "B" the event "first arrival is a non-priority request". This one occurs with the probability: $P(B) = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. In this case, we will have two possibilities:

Therefore, the transition probability $P_{k,i}(i, j)$ is given by:

$$P_{(0,0)}(i, j) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \int \frac{(\lambda_1 t)^i (\lambda_2 t)^j}{i! j!} e^{-(\lambda_1 + \lambda_2)t} E_{\mu_1}(t) dt,$$

$$P_{(0,0)}(0, j) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \int e^{-\lambda_1 t} (\lambda_2 t)^j j! e^{-\lambda_2 t} E_{\mu_2}(t) dt,$$ (67)

$$P_{(0,0)}(1, j) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \int \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} \lambda_1 t e^{-\lambda_1 t} E_{\mu_2}(t) dt,$$

$$P_{(0,0)}(i, j) = 0, i \geq 2.$$

References


Using radial basis functions to solve numerically some MTFDEs

Maria Filomena Teodoro

1 CEMAT, Instituto Superior Técnico, Avenida Rovisco Pais, 1, 1048-001 Lisboa, Portugal
2 CINAV, Escola Naval, Base Naval de Lisboa, Alfeite, 1910-001 Almada, Portugal

E-mail: maria.alves.teodoro@marinha.pt

Abstract In applied sciences, many mathematical models show up functional differential equations with delayed and advanced arguments, the mixed type functional differential equations (MTFDEs). In this class of functional differential equations with delay-advanced argument, the derivative of the unknown function depends on itself evaluated in delayed, advanced and on time values of argument. MTFDEs appear in a wide array of different areas of knowledge such as optimal control [12, 13], economic dynamics [14], nerve conduction [3, 4, 5, 9, 16], traveling waves in a spatial lattice [1, 11], aeroelastic oscillatory phenomena [17, 18, 10] which often appear in physiology (e.g. human phonation), quantum photonic systems [2]. We are particularly interested in the numerical approximation of a delay-advance differential equation introduced in [6]. The numerical scheme using radial basis functions to solve the mixed type equation under study is the natural evolution of the work presented in [15, 7, 8].

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References


Keywords: numerical solution; mixed type functional differential equations; radial basis functions; collocation.
2010 Mathematics Subject Classification : 34K06; 34K10; 34K28; 65Q05.


Optimal control problem for evolution equation of fractional order with a weak degeneracy

Marina Plekhanova$^{1,2}$

$^1$South Ural State University, Russia  $^2$Chelyabinsk State University, Russia

E-mail: mariner79@mail.ru

Abstract An optimal control problem is considered for a degenerate nonlinear evolution equation not resolved with respect to the fractional time derivative. The conditions of a control problem solvability with an abstract cost functional are obtained. The cost functional can be taken as compromise functional, as terminal functional and others.

Let $\mathcal{X}, \mathcal{Y}, \mathcal{U}$ be Banach spaces, $L \in \mathcal{L}(\mathcal{X};\mathcal{Y})$ (linear and continuous from $\mathcal{X}$ to $\mathcal{Y}$), $\ker L \neq \{0\}$, $B \in \mathcal{L}(\mathcal{U};\mathcal{Y})$, $M \in \mathcal{C}(\mathcal{X};\mathcal{Y})$ (linear, closed and densely defined in $\mathcal{X}$ with image in $\mathcal{Y}$), $D_{M}$ be a domain of an operator $M$, endowed by the graph norm $\| \cdot \|_{D_{M}} = \| \cdot \|_{\mathcal{X}} + \| M \cdot \|_{\mathcal{Y}}$, $N: (t_{0}, T) \times \mathcal{X} \to \mathcal{Y}$ be a nonlinear operator. Denote $\rho^{L}(M) = \{ \mu \in \mathcal{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathcal{Y};\mathcal{X}) \}, R_{\mu}^{L}(M) = (\mu L - M)^{-1} L$. An operator $M$ is called $(L, \sigma)$-bounded, if

$$\exists \alpha > 0 \quad \forall \mu \in \mathcal{C} \quad (| \mu | > \alpha) \Rightarrow (\mu \in \rho^{L}(M)).$$

Denote $g_{\delta}(t) = \Gamma(\delta)^{-1} t^{\delta-1}$ for $t > 0$, $f_{t}^{\delta} h(t) = \int_{t_{0}}^{t} g_{\delta}(t-s) h(s) ds$ for $t > t_{0} \in \mathbb{R}, \delta > 0$. Let $\alpha > 0$, $m$ is the smallest positive number not exceeding by $\alpha$, $D_{t}^{m}$ is a usual derivative of the order $m \in \mathbb{N}$, $f_{t}^{0}$ is the identical operator, $D_{t}^{a} f(t) = D_{t}^{m} f_{t}^{m-a} \left( f(t) - \sum_{k=0}^{m-1} f^{(k)}(t_{0}) t(t_{0}) g_{k+1}(t-t_{0}) \right)$ is the Caputo derivative.

Consider control problem

$$LD_{t}^{a} x(t) = M x(t) + N(t, x(t)) + Bu(t), \quad t \in (t_{0}, T),$$

$$\left( P x \right)^{(k)}(t_{0}) = x_{k}, \quad k = 0, 1, \ldots, m-1,$$

$$u \in \mathcal{U}_{\delta},$$

$$J(x, u) \to \text{inf},$$

where $\mathcal{U}_{\delta}$ is an admissible controls set (nonempty convex closed subset of the controls space), $J$ is a cost functional. Initial conditions [69] is called the generalized Showalter — Sidorov conditions. Here operator $M$ is $(L, \sigma)$-bounded, $P = \frac{1}{2\pi i} \int_{|\mu|=\alpha+1} R_{\mu}^{L}(M) \ d \mu$ is a projector along the degeneracy subspace. Put $\mathcal{X}^{1} = \text{im}P$.

A strong solution of problem [68], [69] is a function $x \in L_{q}(t_{0}, T; D_{M}) \cap C^{m-1}([t_{0}, T]; \mathcal{X})$, such that

$$J_{t}^{m-a} \left( x(t) - \sum_{k=0}^{m-1} x^{(k)}(t_{0}) g_{k+1}(t-t_{0}) \right) \in W_{q}^{m}(t_{0}, T; \mathcal{X}),$$

equality [68] is valid almost everywhere on $(t_{0}, T)$, and equality [69] is true.

Keywords: optimal control problem; fractional differential equation; degenerate evolution equation; solvability of a control problem.

2010 Mathematics Subject Classification: 49J20; 34G20; 47J35; 35R11; 34A08.
Control problem (68)–(71) with $\alpha = 1$ is considered in [1]. Solvability of initial problem (68), (69) with $\alpha > 0$ is studied in [2].

Taking into account the form of equation (68) solutions will be found in the Banach space

$$Z_{a,q} = \left\{ x \in L_q(t_0, T; D_M) \cap C^{m-1}([t_0, T]; \mathcal{X}) : f^m_{-a} \left( x - \sum_{k=0}^{m-1} x^{(k)}(t_0) \hat{g}_{k+1} \right) \in W^m_q(t_0, T; \mathcal{X}) \right\}, \quad q > 1,$$

with the norm

$$\|x\|_{Z_{a,q}} = \|x\|_{L_q(t_0, T; D_M)} + \|x\|_{C^{m-1}([t_0, T]; \mathcal{X})} + \|D^a_t x\|_{L_q(t_0, T; \mathcal{X})}.$$  

Here $\hat{g}_k(t) = g_k(t - t_0)$. Set of pairs $(x, u)$ is the set of admissible pairs $\mathcal{W}$ of problem (68)–(71), if $x \in Z_{a,q}(t_0, T; \mathcal{X})$ is a strong solution of problem (68), (69) with $u \in \mathcal{U}_0$ and $f(x, u) < \infty$. The problem (68)–(71) is the finding of pairs $(\hat{x}, \hat{u}) \in \mathcal{W}$ such that $f(\hat{x}, \hat{u}) = \inf_{(x, u) \in \mathcal{W}} f(x, u)$.

**Theorem 22.** Let $\alpha, q > 1$, operator $M$ be $(L, \sigma)$-bounded, $B \in \mathcal{L}(\mathcal{U}; \mathcal{Y})$, a mapping $N \in C([t_0, T] \times \mathcal{X}; \mathcal{Y})$ be locally Lipschitz continuous in $x \in \mathcal{X}$ uniformly with respect to $t \in [t_0, T]$, $x_k \in \mathcal{X}^1$, $k = 0, 1, \ldots, m - 1$, $\mathcal{U}_0$ be a nonempty convex closed subset of $L_q(t_0, T; \mathcal{Y})$. Let for some $u_0 \in \mathcal{U}_0$ there exist a strong solution of (68), (69); a set $Z_{a,q}$ be continuously embedded in a Banach space $\mathcal{Y}$, which is continuously embedded in $L_q(t_0, T; \mathcal{X})$, a cost functional $J$ be convex, bounded from below and lower semicontinuous on $\mathcal{W} \times L_q(t_0, T; \mathcal{Y})$, coercive on $Z_{a,q} \times L_q(t_0, T; \mathcal{Y})$. Then there exists a solution $(\hat{x}, \hat{u}) \in Z_{a,q} \times \mathcal{U}_0$ of problem (68)–(71).

**Proof.** Denote spaces $\mathcal{Y}_1 = Z_{a,q}$, $\mathcal{Y} = L_q(t_0, T; \mathcal{Y}) \times \mathcal{X}^m$, $\mathcal{U} = L_q(t_0, T; \mathcal{U})$ and operators

$$\mathcal{G}(x, u) = -(N(x, x), x, x_0, \ldots, x_{m-1}),$$

$$\mathcal{L}(x, u) = \left( LD^a_t x - Mx - Bu, \gamma_0(Px), \gamma_0([Px]^{(1)}), \ldots, \gamma_0([Px]^{(m-1)}) \right).$$

Here operator $\gamma_0 : C([t_0, T]; \mathcal{Y}) \to \mathcal{Z}$, $\gamma_0 x = x(t_0)$. It is continuous operator on $Z_{a,q}(t_0, T; \mathcal{X})$. The operator $\mathcal{L} : \mathcal{Y}_1 \times \mathcal{U} \to \mathcal{W}$ is linear and continuous. For $v^* \in (L_q(t_0, T; \mathcal{Y})^*)^*$ conditions on operator $N$ give the continuous extension existence of the functional $w(\cdot) = v^*(\mathcal{G}(\cdot))$ from $Z_{a,q}$ on $L_q(t_0, T; \mathcal{X})$. By Theorem 1.2.4 [3] problem (68)–(71) has a solution. \qed

**References**


Symmetry approach for investigation dynamics of two-phase medium

Aleksandr Panov
Chelyabinsk State University, Russia
E-mail: gjd@bk.ru

Abstract A system of two-phase medium is investigated. Lie algebra of symmetry group of this system was found. Different solutions with respect to 4-dimentional subalgebras were found. Some solutions describe instant source in two-phase medium.

Introduction

There is considered a system of partial differential equations, which describes dynamics of two-phase medium[1]

\[
\begin{align*}
\frac{d\rho_1}{dt_1} + \rho_1 \text{div } \vec{u}_1 &= 0, \\
\frac{d\rho_2}{dt_2} + \rho_2 \text{div } \vec{u}_2 &= 0, \\
\rho_1 \frac{d\vec{u}_1}{dt_1} + m_1 \nabla P(\rho_1, \rho_2) &= -\frac{\rho_2}{\tau} (\vec{u}_1 - \vec{u}_2), \\
\rho_2 \frac{d\vec{u}_2}{dt_2} + m_2 \nabla P(\rho_1, \rho_2) &= \frac{\rho_2}{\tau} (\vec{u}_1 - \vec{u}_2).
\end{align*}
\]

Here unknown functions are vectors of velocities \(\vec{u}_1, \vec{u}_2\) and phase densities \(\rho_1, \rho_2\). Pressure \(P\) and volume concentrations \(m_1, m_2\) are some functions of densities and \(\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}_1 \cdot \nabla, \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}_2 \cdot \nabla\) are Lagrange derivatives.

Main results

Lie algebra of symmetry group of this system was found[2]. Optimal systems of subalgebras all dimensionals for this algebra were found in papers [3][4]. Each subalgebra from these optimal systems provides dissimilar invariant, partially invariant, or differential invariant submodel [5]. Solutions of this submodels are called invariant, partially invariant, or differential invariant solutions of original system.

Four-dimensional subalgebras from optimal systems of subalgebras give partially invariant or invariant solutions. All invariant solutions for 4-dimensional subalgebras are found. Also partially invariant solutions with respect to 4-dimensional subalgebras are found. Some of these solutions describe instant source of gaz and particles in \(\mathbb{R}^3\).

Keywords: symmetry group, two-phase medium, invariant solution, instant source.

2010 Mathematics Subject Classification : 35B06; 35K58; 35K70.
References


On reliable and efficient method for the soliton solutions of the coupled Schrödinger–Boussinesq equations

Anwar Ja’afar Mohamad Jawad

Al-Rafidain University College, Baghdad-00964, Iraq

E-mail: anwar_jawad2001@yahoo.com

Abstract In this paper, the coupled Schrödinger–Boussinesq equations SBE will be solved by the Sech, Tanh, Csch, and the modified simplest equation method (MSEM). We obtain exact solutions of the nonlinear for bright, dark, singular 1-soliton solution. Kerr law nonlinearity media are studied. Results have proven that modified simplest equation method is reliable and efficient in handling nonlinear problems. Solutions may find practical applications and will be important for the conservation laws for dispersive optical solitons.

References


Keywords: Nonlinear PDEs; The coupled Schrödinger–Boussinesq equations; exact Solutions; Sech-Tanh Csch function methods.

2010 Mathematics Subject Classification: 35A20; 35A16; 35A25
On unique solvability of a system of equations with memory effects

Lidiya Borel

Chelyabinsk State University, Russia

E-mail: lidiya904@yandex.ru

Abstract An initial boundary value problem is considered for the system of gravitational-gyroscopic waves in Boussinesq approximation. The existence of a unique solution for the problem is proved by methods of theory of degenerate evolution equations in Banach spaces.

Introduction

An initial boundary value problem is considered for the system of equations that describing in Boussinesq approximation the small oscillations of an ideal incompressible fluid uniformly rotating relative to the vertical axis in the field of gravity. It is called the system of gravitational-gyroscopic waves in Boussinesq approximation. The system is unsolvable with respect to the time derivative since it does not contain derivatives with respect to time of some unknown functions, so it is degenerate evolution system. Besides, it contains Volterra integral operator, therefore it is the equations system with memory effects.

Using general approach for the degenerate evolution equations with memory in Banach spaces [1], it is proved the existence of a unique solution of an initial boundary value problem for the system.

References


Keywords: gravitational-gyroscopic waves system; Boussinesq approximation; equation with memory.

2010 Mathematics Subject Classification: 34G10; 35R09.
Numerical solution of fractional order vibration equation using Bernstein polynomial

Vivek Mishra¹, Subir Das²

¹TCET Mumbai, India  ²IIT(BHU), Varanasi India

E-mail: ¹vivmish2010@gmail.com; ²sdas.apm@iitbhu.ac.in

Abstract  In the present article, an effort has been given to solve fractional order vibration equation with the help of operational matrix with Bernstein polynomial basis. Numerical calculations are carried out for integer order as well as fractional order derivatives for different particular cases, which are depicted through graphs.

References


Keywords: Vibration equation; Bernstein Polynomial; Operational Matrix; Fractional order.

2010 Mathematics Subject Classification: 34A08, 74H45, 26A33, 49Mxx, 44Axx.
Solving boundary value problems using radial basis functions networks learned by trust region method

Vladimir I. Gorbachenko¹, Maxim V. Zhukov², Mohie M. Alqezweeni³

Penza State University, Russia

E-mail: ¹gorvi@mail.ru; ²maxim.zh@gmail.com; ³mohieit@mail.ru

Abstract The method using radial basis function networks (RBFN) to solve boundary value problems of mathematical physics is presented in this paper. The main advantages of meshfree methods based on RBFN are explained here. To learn RBFNs, Trust Region Method (TRM) is proposed. It simplifies the process of networks structure selection and reduces time expenses to adjust their parameters. At each step of an error functional minimization the Hessian matrix is replaced by the result of two Jacobi matrices multiplication, that leads to the solution of the conditional problem of a quadratic functional minimization. To solve it, the Staibaugh method is used, which is a modification of the Preconditioned Conjugate Gradient Method. Application of the proposed algorithm is illustrated by solving two-dimensional Poisson equation.

Introduction

The meshfree methods of solving boundary value problems have been widely studied in different researches last decade [1]. Meshfree methods belong to the class of projection methods. In this paper, we are studying one of the most promising meshfree method using the radial basis functions (RBF). In the case of their application, the approximation of the decision is presented as the weighted sum of RBFs where weights are selected in such a way that the approximated solution satisfies the boundary value problem in the selected sample points. The major difficulty of using the meshfree methods based on RBFs is the non-formalizable selection of basis functions parameters. This issue can be overcome with radial basis function networks (RBFNs) [2-3]. The process of boundary value problems solution using RBFNs comes down to the RBFNs learning. The main difference between RBFN method and other meshfree methods is that it adjusts not only the weight of the basis functions but also their parameters. The main purpose of this paper is to develop a method of learning RBFNs based on TRM [4], in order to achieve the significant time reduction of the network parameters configuration.

Main results

RBFN is a network consisting of two layers. The first layer realizes non-linear conversion of an input vector. The second layer does the linear summation. RBF is used as a conversion function. The process of boundary value problems solution using RBFNs comes down to the RBFNs learning, i.e. find such values of and that the error functional representing the sum of squares of residuals in sample points reaches the minimum value. The trained network provides the solution at any arbitrary point when its coordinated comes to the input layer of the network. The efficiency of neural network method for solving boundary value problems depends on the efficiency of the method of solving the problem of the error functional minimization. TRM is one of the best approaches to solve the...
problem. The basic idea of TRM is that at each iteration of the function minimization the function is replaced by an approximating function in the trust region and the minimum of is calculated in trust region, which becomes a new minimum of the function. Depending on how the decreasing, predicted by the model is confirmed by the objective function, the decision on the expansion or contraction of the trust region is taken. Since the error functional is twice differentiable function, hence the second order Taylor series can be used as model of error functional. To get the second order Taylor series it is necessary to calculate the Hessian matrix, that has a large computation cost. Instead of the exact value of the Hessian we are using its approximate value, which is a multiplication of the Jacobi matrices. Using the second order Taylor series leads to the need of solving the conditional problem of a quadratic functional minimization. To solve it, Staihaug method [5] is used here. The method is a modification of the method of preconditioned conjugate directions, taking into account, the restrictions on the solution during functional minimization. As an example of solving boundary value problems using RBFN, trained TRM, solved the boundary value problem for the two-dimensional Poisson equation. TRM reduced the training time of the network and reduce the error of the solution.

Acknowledgments

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References


Bayesian bandwidth selection using two multiplicative bias correction estimators for discrete associated kernel

Lynda Harfouche¹, Nabil Zougab², Smail Adjabi³

¹Bejaia University, Algeria

E-mail: ¹lyndaharfouche@hotmail.com; ²nabilzougab@yahoo.fr; ³adjabi@hotmail.com

Abstract This paper proposed two new estimators called MBC techniques in the context of estimating the discrete probability mass function. For the choice of the bandwidth, we proposed the Bayes global method against the unbiased cross-validation method. We used the Bayesian Markov chain Monte Carlo (MCMC) method for deriving the global optimal bandwidth and we have compared the two proposed method. The performance of both methods is evaluated under the integrated square error criterion. The obtained results show that the Bayes global method performs better than cross-validation.

Introduction

Suppose that \( X_1, ..., X_n \) be a random sample from a distribution with an unknown probability mass function \( f(x) \). The kernel estimator \( f(x) \) of \( f(x) \) is defined by

\[
\hat{f}_h(x) = \frac{1}{n} \sum_{i=1}^{n} K_{x,h}(X_i),
\]

where \( K \) is the kernel function and \( h \) is the smoothing bandwidth. Recently the asymmetric kernels have emerged as a viable alternative that can solve the problem of boundary bias. Several papers report favorable evidence from applying them to empirical models in different domains. The objective of this paper is to propose a new estimators called MBC techniques in the context of estimating the discrete probability mass function with kernel method, and to propose the Bayes global method against the unbiased cross-validation method for the choice of smoothing parameter \( h \). The use of MBC techniques has the advantages that improving the order of magnitude of bias of the estimator \( \text{(72)} \) from \( O(h) \) to \( O(h^2) \).

Nonparametric MBC estimators

Following the same idea of [1] and [3] and using the discrete associated kernel, the first class of MBC techniques is a multiplicative combination of two pmf estimators which we denote as \( \tilde{f}_{TS}(x) \). Then, the TS-MBC kernel estimator can be adapted as follows:

\[
\tilde{f}_{TS}(x) = \{\hat{f}_h(x)\}^{1-c} \{\hat{f}_{h/2}(x)\}^{-c},
\]

where \( \hat{f}_h \) and \( \hat{f}_{h/2} \) are the pmf estimators kernel given by formula \( \text{(72)} \) respectively and \( c \) is a constant with \( c \in (0,1) \) which does not depend on the target \( x \), for more details see [3].

Keywords: Multiplicative bias correction; Bayesian global approach; Cross validation; boundary bias; Integrated square error.

2010 Mathematics Subject Classification: 60J57; 62C10; 47L65.
The second approach of MBC techniques for symmetric kernel density estimators is attributed to [3]. Now, using the discrete associated kernel and we denote as \( \hat{f}_{JLN}(x) \) the following estimator:

\[
\hat{f}_{JLN}(x) = \hat{f}_h(x) \left\{ \frac{1}{n} \sum_{i=1}^{n} \frac{K_{x,h}(X_i)}{\hat{f}_h(X_i)} \right\},
\]

(74)

where \( K_{x,h} \) is the kernel.

References


Time-frequency analysis of Particle's trajectory in perturbed elliptic oscillator

Vinay Kumar

Zakir Husain Delhi College, University of Delhi, Delhi, India

E-mail: vkumar@zh.du.ac.in;

Abstract In the present article, we have numerically investigated the nature of motion in Perturbed Elliptic Oscillator using time-frequency analysis (TFA) based on the wavelet transform. TFA based on the wavelet transform generates time-frequency landscape, known as ridge-plot. On the basis of Ridge-plot, we classify the regular and chaotic trajectories in this nonlinear dynamical model. Numerical experiments suggest that the computational time of this ridge-plot is almost negligible as compared to other chaos indicators (such as Lyapunov Characteristic Exponent (LCE), Smaller Alignment Index Method (SALI), etc.). This method requires less computational effort, and it is readily applicable to higher dimensional dynamical systems. The present experiment verifies this aspect. Sometimes to know whether a given trajectory is chaotic it is desirable to know when, where and to what degree an orbit is chaotic. Also, to compute the time interval of resonance trapping, visualization of the phenomenon of transient chaos are some important aspects of the phase space structure. With the help of ridge-plots, we have addressed these pertinent issues. Additionally, the difference between periodic and quasi-periodic, sticky and non-sticky trajectories are presented using ridge-plots. We have also used the method of Poincare surface of section as a supporting tool for getting several initial conditions and the idea about regular and chaotic regions of the phase space of this model.

References


Keywords: Wavelet analysis; perturbed elliptic oscillator; time-frequency analysis.

2010 Mathematics Subject Classification: 8508; 70F05; 70F15.
Invariant solutions of Black — Scholes type equations for futures-style American options traded at Moscow Exchange

Mikhail M. Dyshaev

Chelyabinsk State University, Chelyabinsk, Russia

E-mail: Mikhail.Dyshaev@gmail.com

Abstract  The group structure of Black — Scholes type equation for futures-style American options in an illiquid market is researched. The group of equivalence transformations for this equation was found. After this, four-dimensional Lie algebras are calculated in the cases of two free element specifications. A three-dimensional Lie algebra corresponds to other nonequivalent specifications. Optimal subalgebras systems and corresponding invariant solutions or invariant submodels are found for every Lie algebra.

Introduction

The Black — Scholes model [1] describes effects occurring in a liquid market. However, if the market of derivatives becomes illiquid, the situation is changing. For example, it is so when a large trader appears on the market, or large-nominal option is trading. Also there are feedback effects between the prices of option and underlying asset prices. This effects arises due to the strategies of dynamic hedging. There are several models that describes such effects on the financial market [2, 3]. Here we investigate the equation for the futures-style American options that are traded on the Moscow Exchange in Russia.

The nonlinear generalization of the Black — Scholes equation [1] in the market with an illiquid underlying asset was studied by group analysis methods [4]. For futures-style options it has the form

\[ u_t + \frac{\sigma^2 x^2 u_{xx}}{2(1 - x v(u_x) u_{xx})^2} = 0. \] (75)

Main results

The Lie algebra of the infinitesimal operators of the equivalence groups of equation (75) with the function \( v \neq 0 \) is found:

\[ Y_1 = \partial_t, \quad Y_2 = \partial_u, \quad Y_3 = x \partial_u, \quad Y_4 = x \partial_x + u \partial_u, \quad Y_5 = u \partial_u - v \partial_v. \]

It is shown that only for \( v = 1 \) and \( v = \beta / u_x \) the basic Lie algebra of the equation (75) has additional symmetries to the core of the basic Lie algebras. Using the symmetries of the equation (75), its invariant solutions are found. In particular, for the function \( v = \beta / u_x \), equation (75) has solutions

\[ u(t, x) = Ax + C, \quad u(t, x) = A \left( \ln x + \frac{\sigma^2 t}{2(1 + \beta)^2} \right) + B, \quad A \neq 0, \]

\[ u(t, x) = B e^{-\frac{\sigma^2 (A - 1)^2 t}{2(1 + \beta)^2}} x^A + C, \quad AB \neq 0. \]

Keywords: nonlinear partial differential equation; group analysis; group of equivalency transformations; group classification; nonlinear Black–Scholes equation; pricing options; dynamic hedging; feedback effects of hedging.

2010 Mathematics Subject Classification: 35K55; 76M60.
References


Optimal system, nonlinear self-adjointness and conservation laws for generalized shallow water wave equation

Mustafa Inc\textsuperscript{1}, Abdullahi Yusuf\textsuperscript{1,2}, Aliyu Isa Aliyu\textsuperscript{1,2}, Dumitru Baleanu\textsuperscript{3,4}

\textsuperscript{1}Firat University, Science Faculty, Department of Mathematics, 23119 Elazığ Türkiye
\textsuperscript{2}Federal University Dutse, PMB 7156, Jigawa State Nigeria Department of Mathematics
\textsuperscript{3}Cankaya University, Department of Mathematics, Öğretmenler Cad. 1406530, Ankara Türkiye
\textsuperscript{4}Institute of Space Sciences, Magurele, Bucharest, Romania

E-mail: \textsuperscript{1}minc@firat.edu.tr; \textsuperscript{1,2}yusufabdullahi@fud.edu.ng, \textsuperscript{1,2}aliyu.isa@fud.edu.ng; \textsuperscript{3,4}dumitru@cankaya.edu.tr

Abstract In this article, the generalized shallow water wave equation (GSWW) is studied from the perspective of one-dimensional optimal system and conservation laws (Cl). Some reduction and new exact solutions are obtained and physical interpretations are presented. Some of the solutions obtained involve expression with Bessel and Airy function. The equation is a nonlinear self-adjoint, CL and one dimensional optimal system are obtained using the conservation theorem by Ibragimov.

Introduction

In this work, we use the method of Ibragimov [1-3] to investigate nonlinear self-adjointness, conservation laws and the one-dimensional optimal system of Eq. (1). The GSWW is given by

\[ \Delta = u_{xxx} + au_{x}u_{xt} + bu_{t}u_{xx} - u_{xt} - u_{xx} = 0, \]  

(76)

where \( a \neq 0, b \neq 0 \) are arbitrary constants.

GSWW equation have been studied by different authors using variety of techniques. For example, [4] introduced exact solutions for GSWW by the general projective Ricatti equations method. Periodic wave solution for GSWW by the improved Jacobi elliptic function method was investigated in [5] and many other.

Main results

We obtain one-dimensional optimal system of subalgebras as follows:

\[ X_{3}, \alpha X_{2} + X_{3}, X_{4}, X_{1} + X_{4}, \alpha X_{3} + X_{4}, \text{ and } \alpha X_{1} + \beta X_{2} + X_{3}, \]  

where \( \alpha, \beta \in \mathbb{R} \). Similarity reduction and exact solutions of the obtained one-dimensional optimal system are analyzed and investigated.

The GSWW is nonlinear self-adjoint with the substitution we obtained. We use the obtained substitution to construct the following conserved vectors for the four infinitesmals, namely, \( X_{1}, X_{2}, X_{3} \) and \( X_{4} \) respectively.

\[ C^{1} = \frac{1}{4} F(t) (-2 + 4 bu_{x}) u_{xt} + u_{xxx}, \]
\[ C^{2} = -\frac{1}{4} F(t) (-2 + 4 bu_{x}) u_{xx} + u_{xxxx}. \]

Keywords: optimal system, Cl, infinitesimal generators, and nonlinear self-adjointness.

2010 Mathematics Subject Classification : 90C46; 18A40.
\[ C^1 = \frac{1}{2} (a - 2b) F(t) u_{xt}, \]
\[ C^2 = -\frac{1}{2} (a - 2b) F(t) u_{xx}. \]

\[ C^1 = \frac{1}{4} F(t) \left( u_{tt} (2 - 2au_x) + (4 - 2au_t) u_{xt} - 3u_{xxtt} \right), \]
\[ C^2 = \frac{1}{4} F(t) \left( 2(-1 + au_x) u_{xt} + 2(-2 + au_t) u_{xx} + 3u_{xxtt} \right). \]

\[ C^1 = \frac{1}{4} F(t) \left\{ -8 + 8au_x + 2atu_{tt} (-1 + au_x) - 4atu_{xt} + 2axu_{xt} - 8bu_{xt} - 2a^2 u(x, t) u_{xt} + 4abu(x, t) u_{xt} + 4abu u_{xt} \right. \]
\[ + 4abxu u_{xt} + u_t \left( 8b - 8abu_x + 2a^2 tu_{xt} \right) - au_{xxt} + 3atu_{xxtt} + axu_{xxxt} \left\}, \right. \]
\[ C^2 = -\frac{1}{4} F(t) \left\{ 4 + 4a^2 u^2_x - 2atu_{xt} - 4atu_{xx} + 2axu_{xx} - 8bu_{xx} - 2a^2 u(x, t) u_{xx} \right. \]
\[ + 4abu(x, t) u_{xx} + 2a^2 tu_{tt} u_{xx} + 2au_x (-4 + atu_{xt} + 2bu_{xx}) + 4au_{xx} + 3atu_{xxt} + axu_{xxxt} \right\}. \]

References


A numerical approach for solving variable order differential equations based on Bernstein polynomials

Hossein Jafari\(^1\), Haleh Tajadodi\(^2\), Dumitru Baleanu\(^3\)

\(^1\)Department of Mathematics, University of Mazandaran, Babolsar, Iran.
\(^2\)Department of Mathematics, University of Sistan and Baluchestan, Zahedan, Iran.
\(^3\)Department of Mathematics, Faculty of Art and Sciences, Cankaya University, Ankara, Turkey.

E-mail: \(^1\)jafari@umz.ac.ir \(^2\)tajadodi.h@gmail.com \(^3\)dumitru@cankaya.edu.tr

Abstract  A numerical technique for solving a class of nonlinear variable order differential equations (VODEs) is introduced by using Bernstein Polynomials (BPs). Here we apply the operational matrix of BPs. With this matrix, the main equation is transformed into a system of algebraic equations by expanding the solution as Bernstein polynomials with unknown coefficients. Then, by solving algebraic equations, the numerical solutions are obtained. The results of numerical examples indicate that the proposed method is computationally efficient.

Introduction

The variable order differential equations (VODEs) denotes a generalization of fractional differential equations (FDEs) which was found as very efficient and interesting tool in different scientific fields \([1,2,3,4,5]\). Recently, numerical approximation theory for variable order differential equations started to become an important topic within the fractional calculus. The aim of this paper is to investigate numerically the variable order differential equations by using Bernstein polynomials. We study the following type of nonlinear variable order differential equation:

\[0_D^\kappa y(t) + \lambda_1 y'(t) + \lambda_2 y(t) + \lambda_3 y''(t) y(t) = g(t), \quad y(0) = y_0,\]  

(77)

where \(g(t), y(t) \in L^2[0,1]\) are known and unknown functions, \(\lambda_1, \lambda_2, \lambda_3\) and \(y_0\) are all constants. If \(\lambda_3 = 0\), the equation (77) reduced to linear case. Here \(0_D^\kappa\) is variable order Caputo derivative which is defined below \([1,2,4,5]\).

Definition 3. The variable order Caputo derivative for \(0 < \kappa(t) \leq 1\) is defined as:

\[0_D^\kappa y(t) = \frac{1}{\Gamma(1-\kappa(t))} \int_0^t (s-t)^{-\kappa(t)} y'(s) \, ds.\]  

(78)

Main results

Approximation of function

It is well known that the set of Bernstein polynomials is a complete basis in Hilbert space \(L^2[0,1]\). Therefore, a function \(y(t) \in L^2[0,1]\) could be expanded in terms of BPs as:

\[y(t) = \sum_{i=0}^m c_i B_{i,m} = C^T \varphi,\]

(79)

where \(C^T = [c_0, c_1, \ldots, c_m]\) is called Bernstein coefficients.

Keywords: Variable order differential equations; Bernstein polynomials; Operational matrix.

2010 Mathematics Subject Classification: 26A33; 30C30.
Operational matrix of derivative operator by Bps

In order to numerically solve the considered variable order FDE, we propose to transform both integer and fractional order derivative operators into matrix forms.

\[ 0D_t^{\kappa(t)} \varphi(t) = 0D_t^{\kappa(t)} (A T_m(t)) = A_0 D_t^{\kappa(t)} T_m(t), \]

so

\[ 0D_t^{\kappa(t)} \varphi(t) = A \left[ \begin{array}{c} \Gamma(2) \\
\Gamma(2 - \kappa(t)) \\
\Gamma(m + 1) \\
\Gamma(m + 1 - \kappa(t)) \\
l^{m-\kappa(t)} \\
\end{array} \right]^T = AMA^{-1} \varphi(t) \] (80)

AMA\(^{-1}\) is called the operational matrix for variable order derivative based on the Bernstein polynomials.

Proposed method

Let approximate \( y(t) \) in (77) by (79). Therefore the (77) can be rewrite as

\[ C T_0 D_t^{\kappa(t)} \varphi(t) + \lambda_1 C T D \varphi(t) + \lambda_2 C T \varphi(t) + \lambda_3 C T D^2 \varphi(t) C T \varphi(t) = g(t), \] (81)

Substituting the operational matrices in the above equation we have

\[ C T_0 A M A^{-1} \varphi(t) + \lambda_1 C T A V A^{-1} \varphi(t) + \lambda_2 C T \varphi(t) + \lambda_3 C T (AVA^{-1})^2 \varphi(t) C T \varphi(t) = g(t), \] (82)

by taking the collocation point \( t_i \) defined by \( t_i = \frac{i}{n}, i = 0,1,\ldots,n \) in Eq. (82), we get the system of algebraic equations. After that we also approximate the initial condition with Bps. Finally, to obtain the solution of Eq. (77) under the given condition, by replacing the row of initial condition by the last row of the matrix (82), we can obtain \( C \). Finally we can obtain the numerical solution as \( y(t) = C T \varphi(t) \).

References


A direct approach to the fast one dimensional total variation regularization algorithm

Artyom Makovetskii\textsuperscript{1a}, Sergei Voronin\textsuperscript{1b} and Vitaly Kober\textsuperscript{2c}

\textsuperscript{1}Chelyabinsk State University, Russian Federation \textsuperscript{2}CICESE, Mexico

E-mail: \textsuperscript{a}artemmac@csu.ru; \textsuperscript{b}voron@csu.ru; \textsuperscript{c}vkober@cicese.mx

Abstract Denosing of a one-dimensional signal corrupted by additive white Gaussian noise has numerous applications in communications, control, machine learning, and many other fields of science. A common way to solve the problem is to utilize the total variation (TV) regularization method. Many efficient numerical algorithms have been developed for solving the TV regularization problem in the 1D case. Condat described a fast direct algorithm to compute the processed 1D signal. The Condat’s approach is based on the dual problem. In this paper, we propose a variant of the Condat’s algorithm based on the direct 1D TV regularization problem. The usage of the Condat’s algorithm with the taut string approach leads to a clear geometric description of the extremal function.

Introduction

One of the most known techniques for denosing of noisy signals and images was proposed by Rudin, Osher and Fatemi \cite{1}. This is a total variation (TV) regularization problem. Let $J(u)$ be the following functional:

$$J(u) = \frac{1}{2} \| u - u_0 \|^2 + \lambda TV(u),$$

(83)

where $\frac{1}{2} \| u - u_0 \|^2$ is called a fidelity term and $\lambda TV(u)$ is called a regularization term. Here $u_0$ is an observed signal that is distorted by additive noise $n$,

$$u_0 = v + n.$$ 

(84)

Consider the following variational problem:

$$u_* = \arg\min_{u \in BV(\Omega)} J(u)$$

(85)

where $u_*$ is an extremal function for $J(u)$. Numerical results have shown that TV regularization is quite useful in image restoration. Here we consider a one dimensional TV (1D TV) regularization problem. In \cite{2} Strong and Chan considered the behavior of explicit solutions of the 1D TV problem when the parameter $\lambda$ in (83) is sufficiently small. Recently, Condat \cite{3} proposed explicit solutions of the 1D TV problem as well as a direct fast algorithm for the case of discrete functions. The algorithm is very fast and has complexity of $O(n)$ for typical discrete functions. The Condat’s approach is based on the dual variational problem. In this paper, we propose a variant of the Condat’s approach based on the direct 1D TV regularization problem. The usage of the Condat’s approach with the taut string method \cite{4} leads to a clear geometric description of the extremal function.

Keywords: optimization theory; total variation; exact solutions; signal and image restoration; denoising.

2010 Mathematics Subject Classification: 49K05.
Main results

Let \( u_0 \) be a discrete function \( u_0 = \{u_0^1, ..., u_0^n\} \). For the function \( u_0 \) the problem (83) takes following form:

\[
J(u) = \frac{1}{2} \sum_{i=1}^{n} (u^i - u_0^i)^2 + \lambda \sum_{i=1}^{n-1} |u^{i+1} - u^i|.
\]  

(86)

Theorem 23. The subgradient \( \nabla J(u) \) can be computed by the following way:

\[
\nabla J(u) = \begin{cases} 
(\nabla J(u))^1 = (u^1 - u_0^1) + \lambda \delta^1 \\
(\nabla J(u))^2 = (u^2 - u_0^2) + \lambda \delta^2 - \lambda \delta^1 \\
... \\
(\nabla J(u))^{n-1} = (u^{n-1} - u_0^{n-1}) + \lambda \delta^{n-1} - \lambda \delta^{n-2} \\
(\nabla J(u))^n = (u^n - u_0^n) + \lambda \delta^{n-1}.
\end{cases}
\]

where

\[
\delta^i = \begin{cases} 
-1, \text{ if } u^{i+1} > u_i \\
1, \text{ if } u^{i+1} > u_i \\
\in [-1; 1], \text{ if } u^{i+1} = u_i.
\end{cases}
\]

Corollary 24. The variational problem (86) can be reduced to the taut string problem [4].

Acknowledgments

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References


Neural network parametric modeling in task about porous catalyst processes

T.V. Lazovskaya\(^1\), D.A. Tarkhov\(^2\), A.N. Vasilyev\(^3\), X.V. Skolis, O.D. Borovskaya

Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: \(^1\)tatianala@list.ru; \(^2\)dtarkov@gmail.com; \(^3\)a.n.vasilyev@gmail.com

Abstract We propose an original approach to the construction of the approximate parametric neural network solutions of differential equations. We consider the fundamental modifications of the classical numerical methods. This technique was tested on several different tasks. We compare the results of applying a new and classical neural network approaches.

Introduction

The construction of the mathematical models of phenomena and processes in physical and technical systems is often associated with problems that are difficult to solve using traditional methods. One of such problems is the necessity for the object model building in the case of unfixed state parameters. These parameters can be expressed in the form of equation coefficients, boundary conditions, domain geometric parameters and so on. The problems arise when parameters are given with an error or it is required to optimize an object with respect to some interval parameters. Sometimes the researcher has the object operation empirical observations and wants to define the approximate differential model parameters more exactly. The classical methods of numerical solving of differential equations such as the mesh (grid) or finite elements methods require the special problem solving for each parameter from a quite big set. The asymptotic methods operate only under certain assumptions about the parameters, for example, when they are small. We propose an original method of building the approximate parametric neural network solutions of differential equations [1–7]. In this work, we consider the implementation of this method in the context of building parametric models of heat and mass transfer processes in the plate porous catalyst granule [8]. In addition, the results of such application [1, 6] are compared with the results of the new method of constructing multi-layer solutions of differential equations [1].

Main stages and results

The mathematical problem consists in finding the solution of the boundary problem of the highly nonlinear second-order ordinary differential equation. The above equation includes three parameters. The required solution depends on these parameters as the arguments. We have built the dependence of one and three parameters. In the first neural network approach, the solution is constructed as a linear combination of basis functions, each containing non-linearly incoming generic parameters.

Results have shown both enough good agreements between the results of [8] and an approximate solution obtained by the package Mathematica. The second approach [1] is the fundamental modifications of known numerical methods such as Euler's method. The main idea is to apply these methods on the interval with a variable upper limit. Thus, the result of classical recursion formulas is a function some numerical coefficients of which can vary to allow the best satisfying the different conditions of the problem being solved. When solving the problem considered, we based on a method of Stormer [2]. The peculiarity of the problem is that classical methods are intended for solving the Cauchy problem and not the boundary one.

Keywords: artificial neural network; parametric solution; porous catalyst; numerical methods.
2010 Mathematics Subject Classification: 68T05; 33F05.
Acknowledgments

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References


Mathematical problems of electrodynamics of layered dielectrics

Yury I. Hudak

Moscow Technological University, Russia

E-mail: hudak@mirea.ru

Abstract A system of mathematical notions enabling uniform representation and solution of direct, optimization and inverse problems in electrodynamics of layered dielectric media using quasi-periodic functions is proposed.

References


Keywords: Direct problem; optimization; inverse problems.

2010 Mathematics Subject Classification: 46N10; 34A55.
Mathematical and computer modelling of sandwich sheet forming process

Igor A. Brigadnov

Department of Computer Sciences, St. Petersburg Mining University, Liniya 21-2, V.O., St. Petersburg, 199106, Russia

E-mail: brigadnov@mail.ru

Abstract Large elastic-plastic deformations of sandwich composite sheets are computer simulated. Within a framework of the shell theory, the elastic-plastic constitutive relation for transversely anisotropic sandwich sheet taking into account of the Bauschinger effect is discussed. Results of the computer simulation of a hemispherical punch stretching operation are demonstrated.

References


Keywords: Composite Sandwich Sheet; Large Elastic-Plastic Deformations; Constitutive Relation; Hill Yield Function; Hemispherical Punch Stretching Process.

2010 Mathematics Subject Classification: 74C15; 74E10; 74K20; 74S05; 74S20.
Mathematical modelling of metal ablation by femtosecond laser pulses

Roman Davydov

1 Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: davydovroman@outlook.com

Abstract In this work, we present a mathematical model for computer simulation of metal ablation by femtosecond laser pulses. The simulation results are compared with experimental data for several metals (Al, Au, Cu, Ni). A good agreement for numerical results with experimental data shows that this model can be employed for choosing laser parameters to receive better accuracy in simulation of metal ablation.

Introduction

The rapid development of ultra-short lasers over the past decades make them very useful tool for laser materials processing. A great deal of progress in nanofabrication of materials has relied on the use of them. Production of nanoparticles can be done in several ways, one of them is laser ablation [1]. Despite extensive research work, accurate prediction of the ablation process is still lacking, because it significantly depending on laser parameters, surrounding medium and target material characteristics. Furthermore, to analyze the physical processes at high energy densities, when laser is used, an adequate description of the thermodynamic properties of matter over a broad region of states including the normal conditions and plasma at high pressures and temperatures is required. Nowadays a two-temperature model has been widely employed for solving ultra-short laser processing of metals [2, 3, 4, 5]. This model describes the energy transfer inside a metal with two coupled generalized heat conduction equations for the temperatures of the electrons and the lattice. To describe the material removal processes it is often inserted into a hydrodynamic code. But the choice of equations of state for materials, required for solving hydrodynamic equations, can significantly affect the results.

We describe the evolution of material parameters using the conservation of mass, momentum and energy of electron and ion subsystems in a two-temperature single-fluid 1D Lagrangian form. To solve the system of hydrodynamic equations in the two-temperature model we construct wide-range semi-empirical two-temperature equation of state. In this equation, a metal is expected to consist of the same electrically neutral atomic cells with atomic weight A and charge Z. To describe thermodynamic properties (such as pressure, temperature and internal energy) we use the Helmholtz free energy. For the single atomic cell, it is proposed as the sum of three terms, describing the electronic and ionic components, and the interaction between them.

Main results

We compare simulation results (ablation depth) after a single laser shot with experimental data for metals. Ultrashort laser pulses are generated by an amplified all-solid-state Ti:Saphire laser chain [6]. Low energy pulses are extracted from a mode-locked oscillator. The pulses are then injected into an amplifying chain including: an optical pulse stretcher, a regenerative amplifier associated with a two-pass amplifier using a 20 W Nd:YLF laser.
as pumping source, and a pulse compressor. Linearly polarized pulses with wavelength centered on 800 nm and typical duration of 100 fs were delivered.

The results of simulation are close to experimental data for all metals in the range 0.1 to 10 J/cm². A good agreement for numerical results with experimental data shows that this mathematical model can be employed for choosing laser parameters to receive better accuracy in simulation of metal ablation.

References


Limit Cycles of the Kukles Cubic System

Valery A. Gaiko

United Institute of Informatics Problems, National Academy of Sciences of Belarus, Belarus

E-mail: valery.gaiko@gmail.com

Abstract Using our bifurcational geometric approach and the Wintner–Perko termination principle, we solve the problem on the maximum number and distribution of limit cycles in the Kukles system representing a planar polynomial dynamical system with arbitrary linear and cubic right-hand sides and having an anti-saddle at the origin.

Introduction

We study the Kukles cubic system

\[ \dot{x} = y, \quad \dot{y} = -x + \delta y + a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x^3 + a_5 x^2 y + a_6 xy^2 + a_7 y^3. \]  

(87)

I. S. Kukles was the first who began to study (87), solving the center-focus problem for this system in [1]. In [2], we constructed a canonical cubic dynamical system of Kukles type and carried out the global qualitative analysis of a special case of the Kukles system corresponding to a generalized cubic Liénard equation. In particular, it was shown that the foci of such a Liénard system could be at most of second order and that such system could have at most three limit cycles in the whole phase plane. Moreover, unlike all previous works on the Kukles type systems, global bifurcations of limit and separatrix cycles using arbitrary (including as large as possible) field rotation parameters of the canonical system were studied. As a result, a classification of all possible types of separatrix cycles for the generalized cubic Liénard system was obtained and all possible distributions of its limit cycles were found.

Main results

Applying Erugin’s two-isocline method [3] and studying the rotation properties of the parameters of (1), we prove the following theorem.

**Theorem 1.** Kukles system (1) with limit cycles can be reduced to the canonical form

\[ \dot{x} = y, \quad \dot{y} = q(x) + (\alpha_0 - \beta + \gamma + \beta x + a_2 x^2) y + (c + d x) y^2 + \gamma y^3, \]  

(88)

where

1) \( q(x) = -x + (1 + 1/a) x^2 - (1/a) x^3, \ a = \pm 1, \pm 2 \) or
2) \( q(x) = -x + b x^3, \ b = 0, -1, \) or
3) \( q(x) = -x + x^2; \)

\( \alpha_0, \ a_2, \gamma \) are field rotation parameters and \( \beta \) is a semi-rotation parameter.

Keywords: planar polynomial dynamical system; Kukles cubic system; field rotation parameter; bifurcation; limit cycle; Wintner–Perko termination principle.

2010 Mathematics Subject Classification: 34C05; 34C07; 34C23; 37G05; 37G10; 37G15.
Using system (88) and studying global bifurcations of its limit cycles, by means of our bifurcational geometric approach, we prove the following theorem.

**Theorem 2.** Kukles cubic system (1) can have at most four limit cycles in (3:1)-distribution.

For the global analysis of limit cycle bifurcations in [3], we used the Wintner–Perko termination principle which connects the main bifurcations of limit cycles. Let us formulate this principle for the polynomial system

\[ \dot{x} = f(x, \mu), \quad (89) \]

where \( x \in \mathbb{R}^2 \); \( \mu \in \mathbb{R}^n \); \( f \in \mathbb{R}^2 \) (\( f \) is a polynomial vector function).

**Theorem 3 (Wintner–Perko termination principle).** Any one-parameter family of multiplicity-m limit cycles of relatively prime polynomial system (89) can be extended in a unique way to a maximal one-parameter family of multiplicity-m limit cycles of (89) which is either open or cyclic. If it is open, then it terminates either as the parameter or the limit cycles become unbounded; or, the family terminates either at a singular point of (89), which is typically a fine focus of multiplicity \( m \), or on a (compound) separatrix cycle of (89), which is also typically of multiplicity \( m \).

Using Theorem 3, we give an alternative proof of Theorem 2 for system (87), namely, we prove the following theorem.

**Theorem 4.** There exists no system (87) having a swallow-tail bifurcation surface of multiplicity-four limit cycles in its parameter space. In other words, system (87) cannot have either a multiplicity-four limit cycle or four limit cycles around a singular point, and the maximum multiplicity or the maximum number of limit cycles surrounding a singular point is equal to three. Moreover, system (87) can have at most four limit cycles with their only possible (3:1)-distribution.

**Acknowledgments**

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**References**


New possibilities of application of artificial intelligence methods for high-precision solution of boundary value problems

Leonid N. Yasnitsky.
Perm State University, Russia
E-mail: yasn@psu.ru

Abstract The problem of estimating the accuracy of boundary value problem solving by numerical methods is discussed. As an alternative to traditional numerical methods, a symbiosis of the Trefftz method and artificial intelligence methods is proposed.

Introduction

Now, one of the most powerful and popular computer simulation tools is modeling based on solving boundary-value problems of mathematical physics. In the modern software market, there is a lot of computer programs implementing numerical methods for solving boundary-value problems of heat conduction, hydrodynamics, elasticity theory, the theory of electric, magnetic, gravitational and even torsion fields. These packages (ANSYS, KOSMOS, WINMASHIN, etc.) are equipped with convenient service and graphics so that any user can easily get acceptable from the point of view of "common sense" approximate solution of almost any boundary-value problem. However, it is very difficult to estimate how much the solution obtained differs from the exact solution of the boundary value problem. As a result, in modern engineering practice, it is not uncommon to find incorrect technical solutions caused by errors in the computer simulation.

The way out of this situation should be sought in order to completely abandon the numerical methods for solving boundary value problems and apply only those methods that lead to exact analytical solutions. But only mathematicians-analysts of the late 18th to the first half of the 20th centuries could obtain exact solutions of boundary value problems, and only for the simplest regions. Now the school of mathematicians-analysts of the past centuries is largely lost.

We propose and develop the idea that, by modeling the intellect of professional mathematicians, we should teach the computer to obtain exact analytic solutions of boundary value problems in the form of functions that exactly satisfy both differential equations and boundary conditions.

Method of Fictitious Canonical Domains and Perspectives

On the site http://www.pspu.ru/regions/ there is a demonstration prototype of the intelligent system "REGIONS" available for general use. This system, in essence, is an expert system that imitates the creative activity of a professional mathematician (expert), who fulfills the solution of boundary-value problems. The package is based on a little-known analytical method - the method of fictitious canonical domains (FCD). This method, proposed by the author of the article as far back as the early 1970s [1][2], at one time allowed finding exact analytical solutions of a number of important problems in practice [1, 3–5]. However, the FCD method did not get distribution because

Keywords: neural networks, nonlinear optimization, differential equations, inverse problem.
2010 Mathematics Subject Classification: 62M45; 68T05.
of the difficulties of its algorithmization. The fact is that his application requires the intellect of a professional mathematician, which made him inaccessible to a wide circle of engineers.

Now the situation has changed. Firstly, in conditions when technogenic accidents and catastrophes have become more frequent, there is an objective need to develop and apply highly reliable methods of mathematical modeling. Secondly, the FCD method can acquire a "second wind" thanks to the latest achievements in the field of artificial intelligence, which allow completely to shift the intellectual problems of its application to the computer [6].

References


New methods of multilayer semiempirical models in Nonlinear bending of the cantilever

Dmitriy A. Tarkhov¹, Tatyana T. Kaverzneva¹, Valerii A. Tereshin¹, Temir V. Vinokhodov¹, Daniil R. Kapitsin¹, Ildar U. Zulkarnay²

¹ Peter the Great St. Petersburg Polytechnic University, Russia
² Bashkir State University, Russia

E-mail: ¹dtarkhov@gmail.com

Abstract This paper is about a new methodology we developed to build a neuronet models that include differential equations and additional data. The core idea is developing a new class of multilayer models, that gives an additional opportunities for combining the classical and new methods. This approach is tested on the task of nonlinear bending of the cantilever loaded metal pipe. Our research is important for long-time prediction of condition and behavior of building beam and contractive elements of hoisting cranes and other mechanisms.

Introduction

Here we solve a reverse problem in modelling loaded element of metal constructions. First, we made an experiment using a pipe having a mass of 116 grams, 7 sinkers having each a mass of 100 grams and one sinker having a mass of 500 grams. One end of the pipe was fixed. Another end of the pipe was been loaded by different sinkers during the experiment. The Position of this end was been measured in the process of its loading by sinkers. The mathematical model of this process was the equation of big static sag of a thin homogeneous physically linear resilient shaft under a distributed force and a concentrated force. Here the distributed force is the weight of the pipe and the concentrated force comes from sinkers. Experimental data received for each sinker were smoothed by neuronet function where each point of measument indicated an empiric angle.

Methods and results

We used two approaches to solve this problem. In the first approach we built a solution of equations with high level accuracy where the main coefficient was calculated through minimization of the sum of squares of the differences between this solution and the empiric values of the angles in the points of the observation. The second approach consists of modification of the classical method by Stormer for solving simple differentiation equations in according with the approach we had developed earlier. This approach consists in that we construct recurrent formulas for numerical solution of simple differentiation equations for the interval with the variable upper limit. As a result, we obtain a solution in the form of a function. Within this procedure the parameters of the main task automatically become the parameters of that function. Simultaneously, a part of constants of the solution can be looked as variables and formed so that to fit experimental data. As a result of implementing our approaches we receive semi empirical solution that much more accurate fit experimental data.

Keywords: neural networks, nonlinear optimization, differential equations, inverse problem.

2010 Mathematics Subject Classification: 62M45; 68T05; 33F05.
References


The stable sequential Lagrange principles as tools for solving some classes unstable inverse problems of electromagnetic theory

Mikhail I. Sumin, Aleksey V. Kalinin, Alla A. Tiukhtina

Lobachevsky State University of Nizhny Novgorod, Russia

E-mail: kalimmm@yandex.ru

Abstract A parametric convex programming problem with an operator equality constraint and a finite set of functional inequality constraints is considered in a Hilbert space. The Lagrange principles in sequential nondifferential form, which are stable with respect to errors in the initial data, are proved. The possibility of using the stable sequential Lagrange principles for solving inverse problem of final observation for Maxwell equations in quasistationary magnetic approximation is discussed.

Introduction

Being the main tool of the theoretical investigation of constrained optimization problems in ideal situations, where initial data for the problems are precisely specified, the Lagrange principle, in the general case, cannot be directly used for solving of optimization problems, the initial data for which may be known only approximately, that is be specified with error, which can be as fixed finite so and, ideally, infinitely small, i.e. to converge to zero. The natural requirement to study problems in such situations inevitably appears, when we meet with the need to solve these or that practical optimization problems. The principal difficulty, arising here by receiving corresponding “approximate” analogues of the classical optimality conditions in the form of the Lagrange principle and the Kuhn-Tucker theorem, is provided, first of all, by an instability of themselves optimization problems. This instability is also naturally inherited by corresponding classical optimality conditions in the sense, that the defined formal optimal elements to perturbed problems may be arbitrary far by the argument and by the function of initial optimal elements to unperturbed problems at arbitrary small perturbations of the problems.

To overcome problems, related with an instability of the classical optimality conditions, in [1, 2, 3, 4], the approach to obtaining stable with respect to errors in the initial data conditions on minimizing sequences in convex programming problems in a Hilbert space was proposed, the main feature of which are the principal basis on the regularization theory ([5, 1]). The obtaining the stable conditions on elements of minimizing sequences is possible just because of the integration of the purely optimizing methods, which are the basis of the deriving the classical optimality conditions, and the based in the duality theory methods of the regularization ([1, 4, 7]). The major feature of the obtained on this way in [1, 2, 3, 4] conditions are that they are nondifferential character and their structure completely repeats the structure of the classical nondifferential optimality conditions. Simultaneously, due to the application the dual regularization ([1, 4, 7]), their formulation contains the obvious (constructive) instruction how to choose the corresponding sequence of dual variables.

The basis on the concept of the minimizing sequence, i. e. the application the sequential approach together with the method of the dual regularization (see, e. g., [1, 4, 7]), allows to transform the classical optimality conditions for constrained minimization problems into assertions of the sequential nature in terms of minimizing sequences.

Keywords: convex programming; parametric problem; minimizing sequence; Lagrange principle in nondifferential form; duality; regularization; quasistationary electromagnetic fields; inverse problem of final observation.

2010 Mathematics Subject Classification: 49K27; 49K40; 35Q61.
which simultaneously are regularized algorithms for a solution of problems. Thus, the application of the sequential approach together with the ideology of the dual regularization considerably expands a class of optimization problems, which can be directly solved on the base of the classical Lagrangian construction.

In present work we illustrate the possibility of application various versions of the stable sequential Lagrange principles and the Kuhn-Tucker theorems ([1] [2] [3] [4]) to the construction of stable with respect to errors in the initial data algorithms for solving unstable inverse problems of final observation for system of Maxwell's equations in quasistationary magnetic approximation [8]. In other words, for stated inverse problems, which can be reduced to equivalent constrained minimization problems in a Hilbert space with equality operator constraint, we construct the stable sequential Lagrange principles and Kuhn-Tucker theorems.

References


Determination of the Loads Acting on the Hovercraft

Vasily Shabarov\textsuperscript{1}, Fedor Peplin\textsuperscript{1}, Andrey Tumanin\textsuperscript{1}, Dmitry Chekmarev\textsuperscript{2}

\textsuperscript{1}Aerohod, Nizhniy Novgorod, Russia
\textsuperscript{2}Lobachevsky university of Nizhny Novgorod, Nizhny Novgorod, Russia

E-mail: \textsuperscript{1}f-peplin@yandex.ru

Abstract The object of the present study is a hovercraft with the ballonet type board seal. For such a vehicle the mathematical model of the dynamics is presented. This model is capable of predicting the trajectory of the vehicle and the loads acting on the hull.

The object of the present study is a hovercraft with the ballonet type board seal. The ballonets are aimed to damp loads acting on the hull and they provide better buoyancy. So this type of vessels combines the advantages of Surface Effect Ships (SES) and Air Cushion Vehicles (ACV) with the classical skirt.

The numerical modeling of dynamics of moving on a water or rigid surface is important in the design of hovercrafts. Numerical experiments at the design stage makes it possible to evaluate the forces and overloads on the ship during the movement. Depending on this, the design parameters can be adjusted to ensure the safety and comfort of passengers, the controllability and strength of the vessel. Ship dynamics modeling is also used in the development of automatic control systems and simulators for crew training.

It is considered the numerical modeling method ACVDYN, developed in shipbuilding company «Aerohod» [1]. The technique is based on the numerical solution of the differential equations of ship motion in three-dimensional space and the differential equations of pressure change in the air cushion sections.

The dynamics of hovercraft is described by a system of differential equations of ship motion and equations describing the change of pressure in the sections of the air cushion of the ship.

The force vector in the right-hand sides of the equations consists of the following components:
- the thrust created by the engines of the ship;
- force from pressure in the air cushion;
- force from the contact of the flexible guard with the water or hard surface;
- force from external aerodynamic flow around the hovercraft;
- gravity.

The vector of moments in the equations is the sum of the moments created by the above forces.

Differential equations are numerically integrated over time by the Euler method.

The adequacy of the description of the real processes of hovercraft motion with this technique is primarily determined by the correct and effective determination of the forces and moments. To determine them, different numerical methods and algorithms are used. The development of the methodology is associated with the improvement of these methods or their replacement by more accurate ones. This work corresponds to this direction.

Proceeding from the practice of mathematical modeling of the dynamics of real hovercrafts, the greatest difficulty is the modeling of forces and moments from the pressure in the air cushion and from the contact interaction of the flexible guard with the water surface. This is due to the complexity of describing the joint processes of the ship (and its structural elements) movement and water under the moving vessel. First, it is necessary to take into account the deformation of the water surface under the influence of pressure in the air cushion. Secondly, the equation must take into account the dissipation of energy in the surrounding water environment.

Keywords: air cushion vehicle; hovercraft; dynamics.
2010 Mathematics Subject Classification : 97M50.
Approaches to the construction of refined models for the determination of data from two types of external forces are considered. The results of numerical experiments, comparison with the full-scale experiment \[2\] are presented, the effect of modification of the numerical simulation technique is estimated.

References


Abstract Bessel-Struve Equation with Nonlocal Condition

Alexander V. Glushak

Belgorod State National Research University, Russia

E-mail: aleglu@mail.ru

Abstract We find conditions for the unique solvability of nonlocal problems for abstract differential equation of the Bessel-Struve equation. Nonlocal conditions contain Erdelyi-Kober operator.

Introduction

Let $A$ be a closed operator in the Banach space $E$ with the domain $D(A)$, which is dense in $E$. For $k > 0$ on the interval $(0, 1]$ consider differential Bessel-Struve equation

$$u''(t) + \frac{k}{t} (u'(t) - u'(0)) = Au(t). \quad (90)$$

Well-posed initial conditions for Bessel-Struve equation (1) required to assign

$$u(0) = u_0, \quad u'(0) = u_1, \quad u_0, u_1 \in D(A).$$

Main results

We seek the solution $u(t) \in C^2([0, 1], E) \cap C((0, 1], D(A))$ of equation (1), which satisfy the nonlocal integrals conditions

$$\lim_{t \to 1} I_{(k-1)/2, \beta} u(t) = u_2, \quad (91)$$

$$\lim_{t \to 1} I_{k/2, \beta} u'(t) = u_3, \quad (92)$$

where $I_{\nu, \beta}$ is the Erdelyi-Kober operator [1], defined by the equality

$$I_{\nu, \beta} u(t) = \frac{2}{\Gamma(\beta) t^{2(\beta+\nu)}} \int_0^t s^{2\nu+1} (t^2 - s^2)^{\beta-1} u(s) \, ds.$$

Problem (1) – (3) with the nonlocal condition (2), (3) is not well-posed in general case. In this paper we find the conditions for an operator $A$ and an element $u_2, u_3 \in E$, which are sufficient for a unique solvability of the problem, an important role is played by the distribution of zeros of the hypergeometric function

$$\chi_{k+2\beta}(\lambda) = \frac{1}{2} \left( \frac{k + 2\beta}{2} ; \frac{k + 2\beta + 1}{2} , \frac{k + 2\beta}{2} + 1 ; \frac{\lambda}{4} \right).$$

Keywords: nonlocal condition, unique solvability, Bessel-Struve equation, Erdelyi-Kober operator.

2010 Mathematics Subject Classification: 42A38, 44A35, 34B30.
Acknowledgments

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References


Nonlinear Integro-Differential Equation of Convolution Type in Lebesgue Spaces

Sultan Askhabov
Chechen State University, Russia
E-mail: askhabov@yandex.ru

Abstract Employing methods from the theory of maximal monotone operators, we prove existence and uniqueness of the solution for a nonlinear integro-differential equation of convolution type in a real Lebesgue space.

Introduction Various classes of nonlinear integral equations of convolution type on a finite (in the periodic case) and an infinite interval of integration were studied in the monograph [1]. There an essential role was played by the positivity property (in the sense Bochner) of convolution-type operators, a property guaranteed by the nonnegativity condition for the discrete (in the periodic case where the interval of integration is the closed interval \([-\pi, \pi]\)) or the integral (in the case where the interval of integration is the whole real axis or semi-axis) Fourier cosine transform of its kernel. In this work we establish that the convolution integro-differential operator is positive if and only if the Fourier sine transform of the kernel is a nonnegative function on the half-line. By using this result, global theorem on the existence and uniqueness of solution for of nonlinear integro-differential convolution-type equation in the real Lebesgue spaces \(L^p(\mathbb{R})\) are proved using the method of maximal monotone operators [2].

Main results Hereafter we assume that given function \(F(x, u)\) generating nonlinearity in the considered equation is defined for \(x, u \in \mathbb{R}\) and satisfies Caratheodory conditions: it is measurable in \(x\) for each fixed \(u\) and is continuous in \(u\) for almost each \(x\). We denote by \(L^+_p(\mathbb{R})\) the set of all non-negative functions in \(L^p(\mathbb{R})\), while \(F\) stands for the superposition operator (Nemytskii operator) generated by function \(F(x, u)\).

**Theorem.** Let \(1 < p < \infty, f(x) \in L^{p'}(\mathbb{R}), p' = p/(p - 1), \) kernel \(h(x) \in L_1(\mathbb{R})\) and
\[
\int_{-\infty}^{\infty} h(t) \cdot \sin(x t) d t \geq 0, \quad \forall x \in [0, \infty).
\]

If for almost \(x \in \mathbb{R}\) and all \(u \in \mathbb{R}\) the nonlinearity \(F(x, u)\) satisfies the conditions
1). \(|F(x, u)| \leq a(x) + d_1|u|^{p-1}\), where \(a(x) \in L^{p'}(\mathbb{R}), \) \(d_1 > 0;\)
2). \(F(x, u)\) is a nondecreasing function of \(u\) for almost each \(x \in \mathbb{R};\)
3). \(F(x, u) \cdot u \geq d_2|u|^p - D(x), \) where \(D(x) \in L^+_1(\mathbb{R}), \) \(d_2 > 0,

**Keywords:** Nonlinear integro-differential equation of convolution type; positive operator; maximal monotone operator.

2010 Mathematics Subject Classification: 45G05; 45B05.
then nonlinear integro-differential equation

$$F(x, u(x)) + \int_{-\infty}^{\infty} h(x-t) u'(t) \, dt = f(x)$$

has a solution $u(x) \in L_p(\mathbb{R})$ with $u'(x) \in L_{p'}(\mathbb{R})$. This solution is unique if $F(x, u)$ strictly increases in $u$ for almost each fixed $x \in \mathbb{R}$.

**References**


Abstract  \( L_p \) - estimates for scalar products of vector fields are obtained. The application of these estimates to study the properties of solutions of various formulations of initial-boundary value problems for Maxwell's equations in the quasi-stationary magnetic approximation is discussed.

Introduction

A demonstration of coercivity of operators associated with differential operations of vector analysis is the essential stage in the study stationary and nonstationary problems of electromagnetic theory. An important role is played by inequalities combined norms of vector field, its curl and divergence in Lebesgue spaces \([1]-[4]\). A feature of study problems in heterogeneous media is that standard operations of vector analysis applied simultaneously to the functions coupled by substantive relations that do not retain smoothness. In this case, in particular, the result on the embedding of functions in Sobolev spaces does not hold and directly application the estimates for \(L_p\)-norms of function is unfeasible.

We propose approach for solving this problem, consisting in the estimation of scalar products of vector fields \([5, 6]\). With the use of special representations of vector fields the estimates combining \(L_p\)-norms of curl and divergence of functions in bounded domains and the class of appropriate weight inequalities in unbounded domains are obtained.

We demonstrate how results about the well-posedness of various formulations of problems for system of Maxwell's equations in the quasi-stationary magnetic approximation can be justified by using these estimates. The quasi-stationary magnetic approximation or eddy current approximation of Maxwell's equations is used for solving a wide class of applied problems related to the formation of electromagnetic fields \([7,8]\). The need to design and justification of efficient numerical methods for solving problems has led to the fact that the well-posedness of different mathematical formulations of boundary and initial-boundary value problems for eddy current Maxwell's equations currently quite actively studied \([9] - [13]\).

The present work deals with the formulation of time-dependent eddy current problems in bounded and unbounded heterognious domains in terms of fields and using potentials. The relations between various formulations of the problems are explained. Using the inequalities for scalar products of vector fields we study the properties of solutions, in particular, their stabilization as \( t \to \infty \). Questions of numerical realization of algorithms of the problems solving are discussed.

Investigated in the present work properties of the solutions can be used in the study of inverse problems and optimal control problems for the Maxwell's equations in the quasi-stationary magnetic approximation \([14, 15]\).

Keywords: estimations; quasi-stationary electromagnetic fields; initial-boundary value problem; heterogeneous media.

2010 Mathematics Subject Classification: 35K51; 35M10; 35Q60.
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Maxwell equations in the quasistationary magnetic approximation and stable sequential Lagrange principles
On decay rate of nonnegative solutions of singular quasilinear parabolic equations

Andrey Muravnik

1JSC "Concern Sozvezdie", Voronezh, Russia  2RUDN University, Moscow, Russia

E-mail: amuravnik@yandex.ru

Abstract  We consider the Cauchy problem for singular quasilinear parabolic equations arising in various applications, prove the uniqueness of its positive solution, and investigate the long-time behavior of the said solution under the assumption that it exists.

The following Cauchy problem is considered:

\[
\frac{\partial u}{\partial t} = \Delta u + \frac{\beta}{u} |\nabla u|^2 + C(x, t) u, \quad x \in \mathbb{R}^n, \quad t > 0, \quad (93)
\]

\[
u(x, 0) = u_0(x), \quad x \in \mathbb{R}^n. \quad (94)
\]

Nonlinearities of the above kind arise in problems of directed polymers and interface growth (see, e.g., [1], [2]). It is assumed that \(u_0(x)\) is nonnegative, continuous, and bounded in the space \(\mathbb{R}^n\), \(\beta > -1\), and there exists a positive constant \(\alpha\) such that one of the two following inequalities is satisfied:

\[
C(x, t) \leq -\alpha \min \left(1, \frac{1}{|x|^2}\right) \quad (95)
\]

or

\[
C(x, t) \leq -\alpha. \quad (96)
\]

We prove the two following assertions:

- No more than one classical bounded nonnegative solution \(u(x, t)\) of problem (93)-(94) exists.
- If inequality (95) is satisfied, than \(u(x, t) \to 0\) uniformly with respect to \(x\) in any compact subset of the space \(\mathbb{R}^n\) (provided that \(u\) exists); if (the stronger) inequality (96) is satisfied, than there exists a positive constant \(a\) such that the inequality \(|u(x, t)| \leq \sup_{x \in \mathbb{R}^n} u_0(x) e^{-at}\) holds in the half-space \(\mathbb{R}^n \times (0, \infty)\) (provided that \(u\) exists).

References


Keywords: parabolic equations; singular equations; quasilinear terms; long-time behavior; decay rate.

2010 Mathematics Subject Classification: 35K55; 35B40.
On properties of mean values of solutions to linear singular differential equations

Lev N. Lyakhov\textsuperscript{1}, Marina V. Polovinkina\textsuperscript{2}, Elina L. Shishkina\textsuperscript{3}

\textsuperscript{1}Voronezh State University, Russia,
\textsuperscript{2}Voronezh State University of engineering technologies, Russia \textsuperscript{3}Voronezh State University, Russia

E-mail: \textsuperscript{1}lewnya@mail.ru; \textsuperscript{2}polovinka-marina@yandex.ru; \textsuperscript{3}ilina_dico@mail.ru

Abstract We study an approach to the proof of mean-value theorems, based on analysis of a differential operator symbol, extended to cover singular differential equations with the Bessel operator.

Introduction

An approach to the proof of mean-value theorems, based on analysis of a differential operator symbol, was proposed in [2], and was generalized in [3] by the methods that was developed in [4]. We use this approach to cover differential equations with the Bessel operator. For this reason, look at [1], [5] and indicate there.

Let $R_n = \{x = (x', x^n), x'=(x_1, \ldots, x_n), x''=(x_{n+1}, \ldots, x_N), x_1>0, \ldots, x_n>0\}, \gamma = (\gamma_1, \ldots, \gamma_n), (x')^\gamma = \prod_{i=1}^{n} x_i^\gamma_i, \gamma_i > 0$. Let $\beta = (\beta', \beta'')$ be a multi-index with non-negative integer components, $\beta' = (\beta_1, \beta_2, \ldots, \beta_n), \beta'' = (\beta_{n+1}, \ldots, \beta_N)$. Denote by $B_{x'}^\beta$ an operator defined by

$$B_{x'}^\beta u = B_{x_1}^{\beta_1} B_{x_2}^{\beta_2} \ldots B_{x_n}^{\beta_n} u,$$

where $B_{x_i} = B_{x_i; \gamma_i}$ is the Bessel operator acting relative to $x_i$ which is defined by

$$B_{x_i} u = B_{x_i; \gamma_i} u = \frac{\partial^2 u}{\partial x_i^2} + \gamma_i \frac{\partial u}{\partial x_i} = x_i^{-\gamma_i} \frac{\partial}{\partial x_i} \left( x_i^{\gamma_i} \frac{\partial u}{\partial x_i} \right).$$

Let $D_{x'}^{\beta''}$ be an operator defined by

$$D_{x'}^{\beta''} f(x', x'') = \partial^{\beta''} f(x', x'') / \partial x_{n+1}^{\beta_{n+1}} \ldots \partial x_N^{\beta_N},$$

where $|\beta''| = \beta_{n+1} + \cdots + \beta_N$.

We define an operator $P = P(B_{x'}, D_{x''})$ with a symbol $P(-\xi_1^2, \ldots, -\xi_{n+1}^2, -i\xi_{n+1}, \ldots, -i\xi_N)$ together with formal-adjoint one $P^*$ by formulas

$$Pu = \sum_{2|\beta'|+|\beta''|\leq m} b_{\beta'} B_{x'}^{\beta'} D_{x''}^{\beta''} u,$$

$$P^* u = \sum_{2|\beta'|+|\beta''|\leq m} b_{\beta''} B_{x'}^{\beta''} (-D_{x''})^{\beta''}.$$

Definition 4. A distribution $\Phi \in \mathcal{E}'_{ev}(\mathbb{R}^+_N)$ is called an accompanying distribution of the equation

$$Pu = 0,$$

if for any solution $u(x) \in C^\infty(\mathbb{R}^n)$

$$\langle \Phi, u \rangle_\gamma = 0.$$
Main result

Theorem 25. A distribution $\Phi$ is an accompaniment of an operator $P$ if and only if the Fourier-Bessel image $F_{B,Y}|\Phi|\xi) \in E_{\nu}(\mathbb{R}_+^N)$ is divisible by the symbol of the formally adjoint of $P$, i.e., $F_{B,Y}|\Phi|\xi) = \hat{\psi}(\xi) P(-\xi_1^2, \ldots, -\xi_n^2, -i\xi')$, where $\psi(\xi)$ is an entire function.

Theorem 26. If $\Phi$ is an accompanying distribution of an operator $P$, then

$$\Phi_0 = \Phi + \lambda F_{B,Y}^{-1}|\Phi|\xi) \hat{P}^*(\xi)(x)$$

is an accompanying distribution of the operator $P + \lambda$.

Theorem 27. Let an operator $P$ can be factorized in the form $P = P_1P_2$, where $P_1, P_2$, are operators of the form $97$, but of less order. Let $\Phi_i$ be a compactly supported accompanying distribution of the operator $P_i$, $i = 1, 2$. Then $\Phi = \Phi_1 \ast \Phi_2$ is an accompanying distribution of the operator $P$.

Acknowledgments

This work is held in memory of the famous Voronezh scientist Ivan A. Kipriyanov.

References

Creep model identification problem for metal structures

Evgenii B. Kuznetsov¹, Sergey S. Leonov² and Alexander N. Vasilyev³

¹,² Moscow Aviation Institute, 4 Volokolamskoye Shosse, 125993 Moscow, Russia
³ Peter the Great St. Petersburg Polytechnical University, 29 Politechnicheskaya Str, 195251 Saint-Petersburg, Russia

E-mail: ¹kuznetsov@mai.ru; ²powerandglory@yandex.ru; ³a.n.vasilyev@gmail.com

Abstract The paper deals with a parameter identification problem for models describing deformation and fracture processes in metal structures operated under high temperature creep conditions. A system of differential equations of kinetic creep theory is used as constitutive equations of creep fracture. The technique of neural network modeling is proposed for solving the parameter identification problem. The procedure of neural network modeling application is demonstrated by the example of parameters definitions for uniaxial tension model for isotropic steel 45 specimens at creep conditions at various stress levels. Results of modeling agree well with theoretical strain-damage characteristics and experimental data.

References


Keywords: Creep; fracture; damage parameter; artificial neural network; parameter identification problem.

2010 Mathematics Subject Classification: 74R20; 65L09; 74G75.
Optical and electronic properties of semiconducting Sn$_2$S$_3$ and SnZrS$_3$

N. Ben Bellil $^1$, F. Litimein, A. BenKaddour

Laboratoire d’Étude des Matériaux & Instrumentations Optiques,
Faculté des Sciences Exactes, Université Djillali Liabès de Sidi Bel Abbès,
Sidi Bel Abbès, Algeria

E-mail: $^1$nn.nn.phy.chi@gmail.com

Abstract We have performed an ab-initio calculation using the FPLAW method within local density approximation (LDA) implemented in WIEN2k code for orthorhombic (Pnma) structure Sn$_2$S$_3$ and SnZrS$_3$ [1]. In addition, the Engel-Vosko exchange potential EV-GGA approach is also used to improve the electronic properties[2]. Our calculated lattice constants of both materials using generalized gradient approximation as developed by Wu and Cohn are in good agreement with experimental values[3]. For band structure calculations, EV-GGA approach provides good results for band gap value as compared to LDA. The estimated GGA (EV-GGA) band gap values are 0.83 eV (1.12 eV) and 0.776 eV (1.15 eV) for Sn$_2$S$_3$ and SnZrS$_3$, respectively. A significant optical anisotropy is clearly observed in the visible-light region. We also calculated and discussed the electron energy loss spectrum for Sn$_2$S$_3$ and SnZrS$_3$. This is the first quantitative theoretical prediction of optical properties and electron energy loss spectrum, for both compounds.

References


Keywords: sulvanite, DFT, EV−GGA, electronic properties, optical properties.

2010 Mathematics Subject Classification: 78A15; 78A60
Variational methods for the solution of fractional discrete/continuous Sturm–Liouville problems

Ricardo Almeida¹, Agnieszka B. Malinowska², M. Luísa Morgado³, Tatiana Odzijewicz⁴

¹University of Aveiro, Portugal ²Bialystok University of Technology, Poland ³University of Trás-Os-Montes e Alto Douro ⁴Warsaw School of Economics, Poland

E-mail: ¹ricardo.almeida@ua.pt; ²a.malinowska@pb.edu.pl; ³luisam@utad.pt; ⁴tatiana.odzijewicz@sgh.waw.pl

Abstract We prove results concerning fractional discrete/continuous Sturm–Liouville problems. The fractional Sturm–Liouville eigenvalue problem appears in many situations, e.g., while solving anomalous diffusion equations coming from physical and engineering applications. Therefore to obtain solutions or approximation of solutions to this problem is of great importance. Here, we describe how the fractional Sturm–Liouville eigenvalue problem can be formulated as a constrained fractional variational principle and show how such formulation can be used in order to approximate the solutions. Numerical examples are given, to illustrate the method.

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References

Keywords: fractional Sturm–Liouville problem, fractional calculus of variations, discrete fractional calculus, continuous fractional calculus.
2010 Mathematics Subject Classification: 26A33; 49R02; 47A75.
Multiple Solutions for a Fourth Order Critical Problem

Abhishek Sarkar

NTIS, University of West Bohemia, Czech Republic

E-mail: sarkara@ntis.zcu.cz

Abstract  Our aim is to look for the existence of at least two positive (weak) solutions of an inhomogeneous fourth-order Navier boundary-value problem involving critical exponential growth on a smooth bounded domain in $\mathbb{R}^4$, with a parameter $\lambda > 0$. We establish upper and lower bounds for $\lambda$, which determine multiplicity and non-existence of solutions.

Introduction

Let $\Omega \subset \mathbb{R}^4$ be a bounded domain with the boundary $\partial \Omega \in C^{2,\sigma}$ for some $0 < \sigma < 1$. In this context, we study the existence of multiple solutions in $W^{2,2}_N(\Omega) = \{ u \in W^{2,2}(\Omega) : u = 0 \text{ on } \partial \Omega \}$, for the following fourth order Navier boundary value problem

$$(P) \quad \begin{cases} \Delta^2 u = \mu |u|^{p-2}u \ln e^u + \lambda h(x) u, & \text{in } \Omega, \\ u, -\Delta u > 0, & \text{on } \partial \Omega, \\ u = \Delta u = 0 & \text{on } \partial \Omega, \end{cases}$$

where $h \geq 0$ in $\Omega$, $\|h\|_{L^2(\Omega)} = 1$, $\lambda > 0$, $\mu = 1$ if $p > 0$ and $\mu \in (0, \lambda_1(\Omega))$ if $p = 0$. Where $\lambda_1(\Omega)$ and $\phi_1$ denote the first eigenvalue and the corresponding eigenfunction of $\Delta^2$ on $W^{2,2}_N(\Omega)$ respectively with respect to the Navier boundary condition. Note that $\lambda_1 > 0$ and $\phi_1$ is strictly positive in $\Omega$. It is worth mentioning that these types of problems are non-compact in nature, due to the generalization of well known Moser-Trudinger type of inequality into higher order derivatives by Adams.[3]

Main results

**Theorem 28.** There exist positive real numbers $\lambda_* \leq \lambda^*$, with $\lambda_*$ independent of $h$, such that the problem $(P)$ has at least two positive solutions $W^{2,2}_N(\Omega)$ for all $\lambda \in (0, \lambda_*)$ and no positive solution for all $\lambda > \lambda^*$.

**Proof.** Define, $\lambda_* := \mu C_0^{-\frac{p+2}{p-2}} \frac{p+2}{\pi p^2}$ where

$$C_0 := \inf_{u \in \mathcal{A} \setminus \{0\}} \int_{\Omega} (p-2u^2)|u|^{p+2} e^{2u^2} > 0,$$

and

$$\Lambda := |u \in W^{2,2}_N(\Omega) : \int_\Omega |\Delta u|^2 \leq (1 + \zeta) \int_\Omega f'(u) u^2 \}.\]

Then,

$$\inf_{u \in \mathcal{A} \setminus \{0\}} \left\{ \mu \int_{\Omega} (p+2u^2)|u|^{p+2} e^{u^2} - \lambda \int_{\Omega} h u \right\} > 0,$$

**Keywords:** Biharmonic; critical exponent; multiple solutions.

2010 Mathematics Subject Classification: 35J30, 35J40, 35J60.
is true whenever $0 < \lambda < \lambda^*$. Next we study the energy functional related to (P), defined by

$$J(u) = \frac{1}{2} \int_\Omega |\Delta u|^2 - \int_\Omega F(u) - \lambda \int_\Omega h u,$$

where $f(u) = \mu u|u|^p e^{u^2}$ and $F(u) = \int_0^u f(s) \, ds$. Since $J$ is unbounded from below on $W^{2,2}_0(\Omega)$, it is better to study the functional $J$ on the Nehari manifold $\mathcal{M} := \{ u \in W^{2,2}_0(\Omega) : \langle f'(u), u \rangle = 0 \}$. It is relevant to study the fibering map (introduced by Drábek-Pohozaev [2]), $\xi_u : \mathbb{R}^+ \to \mathbb{R}$ defined by

$$\xi_u(s) = s \int_\Omega |\Delta u|^2 - \int_\Omega f(s u) u, \quad \forall u \in W^{2,2}_0(\Omega) \setminus \{0\}.$$

The choice of fibering map comes from the fact that $\xi_u(s) = \lambda \int_\Omega h u$ if and only if $su \in \mathcal{M}$, $\forall s > 0$.

We conclude existence of two positive weak solutions by showing one solution arises as a local minimizer and another one as a Mountain Pass Solution. Also, we define

$$\lambda^* := p \mu^{-\frac{1}{p-1}} \left( \frac{\lambda_1}{p+1} \right)^{\frac{p}{p+1}} \left( \int_\Omega h \phi_1 \right).$$

We prove, there is no solution of (P) when $\lambda > \lambda^*$. Assume, $u_\lambda$ be a solution of (P). Now multiply (P) by $\phi_1$ and then integrating by parts over $\Omega$, we get

$$\int_\Omega \phi_1 (\Delta^2 u_\lambda) = \int_\Omega f(u_\lambda) \phi_1 + \lambda \int_\Omega h \phi_1,$$

which implies

$$\lambda \int_\Omega h \phi_1 = \int_\Omega (\lambda_1 u_\lambda - f(u_\lambda)) \phi_1. \quad (98)$$

We see that, $\lambda_1 t - f(t) \leq \lambda_1 t - \mu t^{p+1} = \Theta(t)$ for all $t > 0$. The global maximum for the function $\Theta$ is $p \mu^{-\frac{1}{p}} \left( \frac{\lambda_1}{p+1} \right)^{\frac{p}{p+1}}$ on $(0, \infty)$. Then, from (98) and the definition of $\lambda^*$, we get $\lambda \leq \lambda^*$. This completes the proof of Theorem 28.

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Study of a dynamic frictional contact problem for hyperviscoelastic material with nonconvex energy density

Mikael Barboteu¹, Leszek Gasiński² and Piotr Kalita²

¹Laboratoire de Mathematiques et de Physique, Université de Perpignan-Via Domitia, France
²Faculty of Mathematics and Computer Science, Jagiellonian University, Poland

E-mail: ¹barboteu@univ-perp.fr; ²{gasinski, kalita}@ii.uj.edu.pl

Abstract We study a dynamic frictional contact problem for hyperviscoelastic material with nonconvex energy density. On the contact boundary we assume the normal compliance law and the generalization of the Coulomb friction law which allows for nonmonotone dependence of the friction force on the tangential velocity. We obtain the existence of a weak solution for the dynamic contact problem with damping and nonconvex stored elastic energy function. The existence result is accompanied by numerical examples which highlight both the efficiency of the numerical modelling used to solve the problem and the lack of uniqueness of solution.

References


Keywords: Dynamics, Hyperelasticity, Viscosity, Normal compliance, Nonmonotone friction, Weak formulation, Time approximation, Existence result, Nonuniqueness, Numerical simulations.

2010 Mathematics Subject Classification: 47J20, 47J35, 49M15, 65K15, 65M60, 74B20, 74D10, 74G15, 74M15.
Strong Approximation of Multiple Ito and Stratonovich Stochastic Integrals

Dmitry E. Kuznetsov

1Peter the Great St.-Petersburg Polytechnic University, Russia

E-mail: sde.kuznetsov@inbox.ru

Abstract It is well known, that Ito stochastic differential equations (SDE) are adequate mathematical models of dynamic systems under the influence of random disturbances. One of the effective approaches to numerical integration of Ito SDE is an approach based on Taylor-Ito and Taylor-Stratonovich expansions. The most important feature of such expansions is presence in them of so called multiple Ito or Stratonovich stochastic integrals, which play the key role for solving the problem of numerical integration of Ito SDE. We successfully use the tool of multiple Fourier series, built in the space $L_2$ and pointwise, for the mean-square approximation of multiple stochastic integrals.

Introduction

Let $(\Omega, F, P)$ be a fixed probability space and $W_t$ — is $F_t$-measurable $\forall t \in [0, T]$ Wiener process with independent components $W_t^i; i = 1, \ldots, m$. Let’s analyze the following Ito SDE:

$$dX_t = a(X_t, t) dt + B(X_t, t) dW_t, X_0 = X(0, \omega),$$

(99)

where $a : \mathbb{R}^n \times [0, T] \to \mathbb{R}^n$, $B : \mathbb{R}^n \times [0, T] \to \mathbb{R}^{n \times m}$ satisfy the standard conditions of existence and uniqueness of strong solution $X_t \in \mathbb{R}^n$ of SDE (99); $X_0$ and $W_t - W_0 (t > 0)$ — are independent. In theorems 1 – 3 we solve the problem of combined mean-square approximation of stochastic integrals from Taylor-Ito and Taylor-Stratonovich expansions for the process $X_t$.

Main results

Theorem 1. Assume, that $\psi_j(\tau) \in C([t, T]) (i = 1, 2, \ldots, k)$ and $|\phi_j(x)|^\infty_{j=0}$ is a full orthonormal system of continuous functions in the space $L_2([t, T])$. Then

$$J[\psi^{(k)}]_{\tau, t} = \lim_{\max |\tau_j| \to \infty} \sum_{j=0}^k \psi_j(t) \cdots \psi_j(t) dW_t^1 \cdots dW_t^k (\text{multiple Ito stochastic integral})$$

where $J[\psi^{(k)}]_{\tau, t} = \int_t^T \psi_t^1(t) \cdots \psi_t^k(t) dW_t^1 \cdots dW_t^k$ (multiple Ito stochastic integral); $\Delta W_{t_j}^{(i)} = W_{t_{j+1}}^{(i)} - W_{t_j}^{(i)} (i = 0, 1, \ldots, m)$, $W_t^{(0)} = \tau$, $\xi_j = \int_t^T \phi_j(\tau) dW_t^j$ are independent standard Gaussian random variables for various $i$ or $j$ if $i \neq j$, $\{\tau_j\}_{j=0}^{N-1}$ — partition of interval $[t, T]$, satisfying the conditions: $t = \tau_0 < \ldots < \tau_N = T$, $\max_{0 \leq j \leq N-1} (\tau_{j+1} - \tau_j) \to 0$ if $N \to \infty$, $C_{j_k, \ldots, j_1} = \int_{t_j}^{t_T} K(t_1, \ldots, t_k) \prod_{i=1}^k \phi_{j_i}(t_i) dt_1 \cdots dt_k$, $K(t_1, \ldots, t_k) = 1_{\{t_1 < \cdots < t_k\}} \psi_1(t_1) \cdots \psi_k(t_k) (t_1, \ldots, t_k \in \mathbb{R})$.

Keywords: multiple Ito stochastic integral; multiple Stratonovich stochastic integral; Taylor-Ito expansion; strong approximation; numerical modeling.

2010 Mathematics Subject Classification: 60H10; 60H05; 60H35; 65C30.
\( \{r, T\} \), \( I_A \) — is an indicator of the set \( A, G_k = H_k \setminus L_k \), \( L_k = \{(l_1, \ldots, l_k) : l_1, \ldots, l_k = 0, 1, \ldots, N-1; l_g \neq l_t (g \neq r); g, r = 1, \ldots, k \}, H_k = \{(l_1, \ldots, l_k) : l_1, \ldots, l_k = 0, 1, \ldots, N-1 \} \).

Let’s consider particular cases of the theorem 1 for \( k = 2, 3, 4 \):

\[
J[\psi(2)]_{T,t} = \sum_{j_1, j_2=0}^{\infty} C_{j_1 j_2} \psi^{(i_1)}_{j_1} \psi^{(i_2)}_{j_2} - 1_{\{i_1=i_2\neq 0, j_1=j_2\}},
\]

\[
J[\psi(3)]_{T,t} = \sum_{j_1, j_2, j_3=0}^{\infty} C_{j_1 j_2 j_3} \psi^{(i_1)}_{j_1} \psi^{(i_2)}_{j_2} \psi^{(i_3)}_{j_3} - 1_{\{i_1=i_2\neq 0, j_3=j_2\}},
\]

\[
J[\psi(4)]_{T,t} = \sum_{j_1=0}^{\infty} (\sum_{j_2=0}^{\infty} C_{j_1 j_2} \psi^{(i_1)}_{j_1} \psi^{(i_2)}_{j_2} - 1_{\{i_1=i_2\neq 0, j_2=j_1\}}) - \sum_{j_2=0}^{\infty} (\sum_{j_1=0}^{\infty} C_{j_2 j_1} \psi^{(i_2)}_{j_1} \psi^{(i_1)}_{j_2} - 1_{\{i_2=i_1\neq 0, j_1=j_2\}}).
\]

Let’s consider the estimates of mean-square errors of approximations, based on theorem 1.

**Theorem 2.** In the conditions of the theorem 1 the following estimates are valid:

\[
M(J[\psi(k)]_{T,t}^{p_1, \ldots, p_k} - J[\psi(k)]_{T,t}^2) \leq K \int_{T-t}^{t} K^2(t_1, \ldots, t_k) dt_1 \ldots dt_k - \sum_{j_1, \ldots, j_k=0}^{p_1, \ldots, p_k} C^{2}_{j_1 \ldots j_k}
\]

\[
(i_1, \ldots, i_k = 0, 1, \ldots, m \text{ and } T-t < 1 \text{ or } i_1, \ldots, i_k = 1, \ldots, m \text{ and pairwaise different},
\]

where \( J[\psi(k)]_{T,t}^{p_1, \ldots, p_k} \) is a truncated series from the theorem 1 with upper limits \( p_1, \ldots, p_k \).

The following theorem is adapt theorem 1 to the multiple Stratonovich stochastic integrals.

**Theorem 3.** Let function \( \psi_1(s) \) is continuously differentiated at \( [t, T] \) and functions \( \psi_1(s), \psi_3(s) \) are two times continuously differentiated at \( [t, T]; \phi_i(x)_{0}^{\infty} = \) a full orthonormal system of Legendre polynomials or trigonometric functions in the space \( L_2([t, T]) \). Then

\[
J^*[\psi(2)]_{T,t} = \int_{\infty}^{t} \int_{\infty}^{t} \psi_1(t_1) dt_1 \ldots dt_k dW_{t_1}^{(i_1)} \ldots dW_{t_k}^{(i_k)} \text{ (multiple Stratonovich stochastic integral); } k = 3, 4 \text{ (for } k = 3: i_1, i_2, i_3 = 1, \ldots, m; \text{ for } k = 4: i_1, \ldots, i_4 = 0, 1, \ldots, m \text{ and } \psi_1(r), \ldots, \psi_4(r) \equiv 1); \text{ the meaning of notations from theorem } 1 \text{ is remained.}
\]

**References**

Inference for stochastic diffusion model for the dynamic of HIV using a Bayesian approach

Abdellah Abou-Bakre\textsuperscript{1}, Hamid El Maroufy\textsuperscript{2}

\textit{FST Sultan Moulay Slimane University, Beni Mellal, Morocco}

E-mail: \textsuperscript{1}a.aboubakre@usms.ma; \textsuperscript{2}h.elmaroufy@usms.ma

**Abstract** We consider a model of the HIV dynamic in an heterosexual population of fixed size $N$. We formulate stochastic diffusion approximation process associated to the discrete model using Fokker-Plank equations. Our aim is to estimate the parameters of this model. Due to low frequency and discrete observations, to reach our goal, we use Bayesian inference with MCMC simulations.

**Model description**

We consider a closed and mixed heterosexual population of size $N$. In which we suppose that the infection can be made only by heterosexual contact as illustrated in the figure 1 where $S_F(t), I_F(t), S_M(t), I_M(t)$ and $Z(t)$ denotes respectively, the sizes of susceptible females, infected females, susceptible males, infected males, and AIDS cases at time $t$.

![Figure 1: Transmission of HIV in closed heterosexual population.](image)

**Inference for non linear diffusion models**

Let consider the inference for an Itô diffusion process of type

$$dX_t = \xi(X_t, \theta)dt + \Sigma^1(X_t, \theta)dW(t)$$

\textbf{Keywords} : Bayesian inference; SIR epidemic model; Diffusion Approximations; Parameter Estimation.

\textbf{2010 Mathematics Subject Classification} : 62P10; 62M05; 62M86; 62F15; 60J60.
The conditions under which the SDE (100), can be solved are well verified. We assume that the process \( X = (x_1(t), x_2(t), y_1(t), y_2(t)) \) will be observed at a finite integer of times. The purpose is to inference for the (unknown) parameter vector \( \theta \) on the basis of partial and discrete observations on \( X(t) \). In practice, it is necessary to work with the discretized version of (100), given by the Euler approximation,

\[
\Delta X(t) = \xi(X(t), \theta) \Delta t + \sum \frac{1}{2} (X(t), \theta) \Delta W(t).
\]

In order to perform inference on the model parameter \( \theta \), one tries to approximate the true transition density \( p_\theta \) of the diffusion process by the Euler scheme. This is eligible only if inter-observation times of the observed data \( X^{obs} \) are small. Since such a requirement is not fulfilled in our case (Observation of HIV). We augment the data \( X^{imp} \), using Eraker approach [3], by imputing intermediate points between each pair of observations. Furthermore, to inference \( \theta \), a MCMC approach is employed to construct a Markov chain \( \{\theta^{(i)}, X^{imp}_{i=1,...,L}\} \) of length \( L \) whose elements are samples form joints posterior density \( \pi(\theta, X^{imp} | X^{obs}) \) of parameter \( \theta \) and imputed data \( X^{imp} \) conditional on \( X^{obs} \) (observations). The Markov chain \( \{\theta^{(i)}\}_{i=1,...,L} \) is regarded as a draw from the marginal density \( \pi(\theta | X^{obs}) \). To construct the Markov chain \( \{\theta^{(i)}, X^{imp}_{(i)}\}_{i=1,...,L} \) we base on two steps:

**Step 1: path update** Draw \( X^{imp}_{(i)} \sim \pi(X^{imp}_{(i)} | X^{obs}, \theta^{(i-1)}) \),

**Step 2: parameter update** Draw \( \theta^{(i)} \sim \pi(\theta^{(i)} | X^{obs}, X^{imp}_{(i)}) \).

However, direct sampling is not possible neither from \( \pi(X^{imp}_{(i)} | X^{obs}, \theta^{(i-1)}) \) nor \( \pi(\theta^{(i)} | X^{obs}, X^{imp}_{(i)}) \). Hence in both steps, a Metropolis Hastings M-H algorithm is used [1, 3].

References


A numerical solution of a moving boundary problem

Ajay Kumar, Abhishek K. Singh, Rajeev*

Department of Mathematical Sciences, IIT(BHU), Varanasi, India

E-mail: *rajeev.apm@iitbhu.ac.in

Abstract In this paper, we consider a moving boundary problem that includes a nonlinear diffusion equation with a nonlinear condition on moving interface. This problem arises when a viscous fluid spreads under the action of gravity above a smooth horizontal surface. For the solution of the problem, we first covert the governing partial differential equation into an ordinary differential equation by scaling and similarity variables. Using the same scaling and similarity variables, the boundary conditions and interface conditions are also changed into the conditions of obtained ordinary differential equation. Then a numerical solution of the modified problem is obtained by using tau method which is based on shifted Chebyshev operational matrix of differentiation. For accuracy of our procedure, a comparison between our result and exact solution for a limit case is depicted through table. This comparison shows that our results are approximately accurate with exact. The dependence of moving boundary on various parameters is also discussed.

References


Keywords: moving boundary problem; similarity variables; shifted Chebyshev polynomial; operational matrix of differentiation.

2010 Mathematics Subject Classification: 35R35; 35R37; 35R60; 60J60.
Using fractional differential equations to model some real phenomena

Ricardo Almeida1, Nuno R. O. Bastos1,2, M. Teresa T. Monteiro3

1Center for Research and Development in Mathematics and Applications (CIDMA), University of Aveiro, Aveiro, Portugal
2School of Technology and Management of Viseu, Polytechnic Institute of Viseu, Viseu, Portugal
3Algoritmi R&D Center, University of Minho, Braga, Portugal

E-mail: 1ricardo.almeida@ua.pt; 2nbastos@estv.ipv.pt; 3tm@dps.uminho.pt

Abstract The goal of this work is to show, based on concrete examples and experimental data from several experiments, that fractional differential equations may model more efficiently certain problems than ordinary differential equations. A numerical optimization approach based on least squares approximation is used to determine the order of the fractional operator that better describes real data, as well as other related parameters.

Introduction

In the present work we present several real problems, each one described by an ordinary or system of ordinary differential equations. We then consider the same problem, but modeled by a fractional or system of fractional differential equations, and compare which of the two models are more suitable to describe the process, based on real experimental data. In our present work we deal with the Caputo derivative.

The Caputo fractional derivative of a function \( y : [a, b] \rightarrow \mathbb{R} \) of order \( \alpha > 0 \) is defined by

\[
\frac{C_a D^\alpha t}{y(t)} = \frac{1}{\Gamma(n - \alpha)} \int_a^t (t - s)^{n-\alpha-1} y^{(n)}(s) ds,
\]

where \( n = [\alpha] + 1 \). In particular, when \( \alpha \in (0, 1) \), we obtain

\[
\frac{C_a D^\alpha t}{y(t)} = \frac{1}{\Gamma(1-\alpha)} \int_a^t (t - s)^{-\alpha} y'(s) ds.
\]

Main results

Given an ordinary differential equation \( y'(t) = f(t, y) \), we replace it by the fractional differential equation \( \frac{C_a D^\alpha t}{y(t)} = f(t, y) \), with \( \alpha \in (0, 1) \). When we consider the limit \( \alpha \to 1^- \), we obtain the initial one. If we consider \( \alpha \in (1, 2) \) as well, and then take the limit \( \alpha \to 1^+ \), we would get \( y'(t) - y'(0) = f(t, y) \), and this is the reason why we will neglect this case firstly.

**Theorem 29.** Let \( f : [a, b] \times \mathbb{R} \rightarrow \mathbb{R} \) be a continuous function and \( \alpha > 0 \) a real. If \( y_a^\alpha \) is a solution of the system

\[
\begin{aligned}
\frac{C_a D^\alpha t}{y(t)} &= f(t, y), \quad \alpha \in (0, 1) \\
y(a) &= y_a
\end{aligned}
\]

**Keywords:** Fractional calculus, fractional differential equation, numerical optimization.

**2010 Mathematics Subject Classification:** 26A33; 34A08; 90C30.
and if the limit
\[ \lim_{\alpha \to 1} y_\alpha^*(t) := y^*(t) \]
exists for all \( t \in [a, b] \), then \( y^* \) is solution of the Cauchy problem
\[
\begin{cases}
  y'(t) = f(t, y) \\
  y(a) = y_a.
\end{cases}
\]

After computing the solution of the fractional differential equation, which depends on \( \alpha \), we consider the function with fractional order \( \alpha \) on the interval \((0, 2)\) in order to increase the accuracy of the method.

Some distinct problems (e.g. exponential law of population growth and blood alcohol level) are studied in this work. In each case, we compute the values of the parameters that better fit with the given data, and also the error in each of both approaches.

For example, if we consider the exponential law of population growth problem, by the standard approach, we have
\[
N'(t) = (B - M)N(t) = PN(t),
\]
where \( N(t) \) is the number of individuals in a population at time \( t \), \( B \) the birth rate, \( M \) the mortality rate and \( P := B - M \) is the production rate. Here we we assume that \( B \) and \( M \) are constant.

If we consider a fractional approach the problem is model by the fractional differential equation
\[
C^{\alpha}_0 D^\alpha_t N(t) = PN(t), \quad t \geq 0, \ \alpha \in (0, 2).
\]

For our numerical treatment of the problem, we find several databases with the world population through the centuries and decidet to use the one provided by the United Nations , from year 1910 until 2010, consisting in 11 values, where the initial value is \( N_0 = 1750 \). For the classical approach, the production rate is
\[
P \approx 1.3501 \times 10^{-2}
\]
and the error from the data with respect to the analytic solution is given by
\[
E_{\text{classical}} \approx 7.0795 \times 10^5.
\]
But, if we take into consideration our fractional model, for \( \alpha \in (0, 2) \) we obtain that the best values are
\[
\alpha = 1.393298754843208 \quad \text{and} \quad P \approx 3.4399 \times 10^{-3}
\]
with error
\[
E_{\text{fractional}} \approx 2.0506 \times 10^5.
\]

Acknowledgments

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References

Stochastic decomposition for a retrial queue type with server subject to breakdowns, repairs and impatient customers

Zirem Djamilia¹, Mohamed Boualem², Djamil Aissani³

E-mail: ¹ziremdjamila@yahoo.fr; ²robertt15@yahoo.fr; ³lamosbejaia@hotmail.com

¹University of Algiers 3, Algeria ²³University of Bejaia, Algeria

Abstract We study a stochastic decomposition of a batch arrivals unreliable server with general retrial time and impatient customers. Here we assume that customers arrive according to compound Poisson processes. Any arriving batch of primary customers finds the server free, one of the customers from the batch enters into the service area and the rest of them join into the orbit. The primary customers who find the server busy or failed are allowed to balk or are queued in the orbit in accordance with FCFS (first come first served) retrial policy. The customer whose service is interrupted can stay at the server waiting for repair or enter into service orbit. After the repair is completed, the server resumes service immediately if the customer in service has remained in the service position. By using supplementary variables technique, we carry out an extensive analysis of the considered model. In steady state the joint distribution of the server state and queue length is obtained, and some performance measures of the system, such as the mean number of customers in the retrial queue and waiting time. Here we are interested in the stochastic decomposition. Also various performance indices have been examined numerically by taking an illustration.

Introduction

Retrial queueing systems have been successfully applied in telephone switching systems, telecommunication networks and computers competing to gain service from a central processing unit, call centers, inventory systems, etc. A complete description of this system can be found in the monographs of [1]. Analytical treatment to realize an extensive analysis of this model is obtained by supplementary variables technique. In this work we focus for the stochastic decomposition law, for the detail of this model see [2].

Stochastic decomposition

The aim of this section is to study the stochastic decomposition property. The state of the system at time $t$ can be described by the Markov process

$$\{X(t), t \geq 0\} = \{J(t); J^*(t); N(t); \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t)\}$$

A key result in these analyses is that the number of customers in the system under study can be expressed as the sum of two independent random variables, one of which is the number of customers in the $M^X/G/1$ retrial queue with impatient customers, breakdowns and repairs, and the other is the number of customers in the system given that the server is idle or busy serving a customer. In particular, in the context of our system, we have the following results.

Keywords: Retrial queue, Batch arrivals, Breakdowns and repairs, stochastic decomposition.

2010 Mathematics Subject Classification: 90C15; 60K25.
Corollary 30. The generating function of the number of customers in the system is given by

\[
P(z) = \left[\frac{(1 - q + qL_A(\lambda))(1 - z)g(z) + L_A(\lambda)(z - g(z))}{(1 - q + qL_A(\lambda))(1 - z)K(z)g(z) - z(z - g(z)K(z)) + K(z)L_A(\lambda)(z - g(z))} \frac{\lambda p(1 - g(z))K(z) + \lambda z(1 - K(z))}{\lambda p(1 - g(z))zL_A(\lambda)(z - K(z)g(z))} \right] \times \frac{1}{\lambda p(1 - g(z))} P_{00}
\]

\(P(Z)\) can be expressed as convolution of two independent variables like, i.e.,

\[P(Z) = \Phi_0(z) \times \chi(z)\]

where \(\Phi_0(z)\) is the generating function of the system size distribution of the \(M^X/G/1\) system with breakdowns and repairs and impatient customer given by

\[\Phi_0(z) = \frac{P_{00} + P_0(z)}{P_{00} + P_0}\]

and \(\chi(z)\) is the probability generating function of conditional distribution of the number of customers in system given that the server is idle or busy serving a customers or just under repair.

\[\chi(z) = \frac{P_{00} + P_0(z)}{P_{00} + P_0}.
\]

Where The system is empty with probability

\[P_{0,0} = \frac{- (1 - L_A(\lambda))(g'(1) + q - 1) + (1 - K'(1))}{(1 - q + (q - pg')(1) + L_A(\lambda))((1 - L_A(\lambda)) + L_A(\lambda)(1 - k'(1)) \left(\frac{1}{pg'(1)} - \frac{1}{p}\right))}.
\]

The server is idle and the system is nonempty with probability

\[P_0 = \frac{- (1 - K'(1) - g'(1))(1 - L_A(\lambda))}{(1 - q + (q - pg')(1) + L_A(\lambda))((1 - L_A(\lambda)) + L_A(\lambda)(1 - k'(1)) \left(\frac{1}{pg'(1)} - \frac{1}{p}\right))}.
\]

\[P_0(z) = \frac{(z - g(z)K(z))(1 - L_A(\lambda))P_{0,0}}{(1 - q + qL_A(\lambda))(1 - z)K(z)g(z) - z(z - g(z)K(z)) + K(z)L_A(\lambda)(z - g(z))}.
\]

Were

\[K'(1) = \lambda pg'(1)\left[1 + \mu(\gamma_1 + (1 - r)\eta_1)\right] \beta_1.
\]

We get

\[\chi(z) = \frac{zL_A(\lambda)(K(z) - z)\left[(1 - L_A(\lambda))g'(1) - (1 - K'(1))\right]}{[(1 - L_A(\lambda))zg(z)K(z) - z(z - K(z))L_A(\lambda)][L_A(\lambda)(K'(1) - 1)].
\]

References


Minimum energy interpolating curves in fluid environments

Luís Machado

Department of Mathematics
Univ. of Trás-os-Montes e Alto Douro (UTAD)
5001-801 Vila Real, Portugal

Centro de Matemática
Univ. of Minho
4710-057 Braga, Portugal

E-mail: lmiguem@utad.pt

Abstract In path planning problems of autonomous underwater vehicles, it is frequently required to find a mission path that minimizes acceleration and drag forces while the vehicle moves from an initial position to a target position passing through a series of waypoints [2].

Drag is a mechanical force generated by the interaction and contact of a solid body with a fluid (gas or liquid). It is generated by the difference in velocity between the solid object and the fluid and acts in the opposite direction to the motion of the object. Its magnitude, which depends on the density of the fluid, the speed and the size, shape and orientation of the body is, typically, proportional to the square of the speed. In this case, the power needed to overcome the drag force is proportional to the cube of the speed.

In this talk we are interested in determining optimal trajectories of a vehicle moving in a fluid environment that minimize not only the power needed to overcome changes in velocity but also the drag forces. Therefore, a variational problem where the energy functional depends on the acceleration and drag is formulated and the corresponding Euler-Lagrange equations are derived.

As we will see, the presence of the drag term, corresponding to the cube power of the velocity, increases substantially the complexity of the problem even when the geometry of the configuration space is not taken into account and the problem is only formulated on a Euclidean space. In the absence of drag, the problem boils down to the classical interpolation problem, whose solutions are obtained by gluing together geometric cubic polynomials prescribing the given data at the given times [3, 1].

Algebraic integrability properties of the Euler-Lagrange equations in Euclidean spaces have been partially studied in [4] using the theory of Darboux polynomials.

A numerical algorithm to find approximate solutions for this highly nonlinear optimization problem is proposed and some of the numerical results are given.

References


Keywords: interpolation problems; calculus of variations on manifolds; Euler-Lagrange equations.

2010 Mathematics Subject Classification: 34H05; 53A17; 53B05; 58E40; 93C15

Investigation of time series by means of factor analysis

Victor V. Garbaruk\textsuperscript{1}, Victor N. Fomenko\textsuperscript{2}

\textit{1Petersburg State Transport University, Russia} \textit{2Petersburg State Transport University, Russia}

E-mail: \textsuperscript{1}vmkaf@pgups.ru; \textsuperscript{2}vfomenko1943@gmail.com

Abstract

Factor analysis is applied to stochastic processes, data points being considered as input variables. Correlation between data points appears as presence in each of them factors common for the whole process. The common and individual factors are determined by means of maximum likelihood method. A version of discriminant analysis of time series is outlined in the framework of the statistical model suggested. Clustering experimental data is done using test of statistical significance. Two criterion variables are introduced: one of them depends on common factors whereas the other is related to individual factors.

Introduction

An important task arising in using time series is their classification into groups according to a specific criterion. Existence of an appreciable stochastic component in the time series disguising the classifying criterion can appreciably complicate solution of the abovementioned problem. The goal of the present paper is to suggest a method which, in a number of cases, enables one to eliminate the influence of the stochastic factor and more clearly distinguish an interesting feature of the time series. The method is based on factor analysis (see, for example [1, 2]) which allows extracting common components in data points of a time series, so-called common factors. All data points are expressed through common factors. The set of \( n \) random variables fill a domain in \( n \) dimensional space which is segment of a linear manifold with dimensionality equal to the number of common factors. Strong correlation between data points ensures that the number of common factors is not large. In such case the stochastic component in the process under investigation can be removed to a large extent and reliable conclusions can be drawn on whether the time series belongs to a specific class. The present approach to take into account correlation between data points is an alternative to autocorrelation function method. Its advantage is that it does not imply transition into frequency domain and, as a result, loss of clearness. Moreover, non-stationary and stationary processes are treated on the same footing. In conventional approach the time-frequency transform would be necessary for non-stationary processes what causes further difficulties.

Let \( X(t) \) be a random process observed at time moments \( t_i (i = 1, \ldots, n) \). Denote the corresponding process values as \( x_i = X(t_i) \). The notations \( m_i = \mathcal{M} x_i \) and \( \sigma_i = \sqrt{\mathcal{D} x_i} \) stand for means and rms deviations. In accordance with the concept of factor analysis the \( x_i \) variables are represented as follows

\[
x_i = m_i + \sum_{k=1}^{p} a_{ik} f_k + u_i; \quad i = 1, n
\]

where \( f_k \) are non-correlated normalized random quantities (common factors). The coefficients \( a_{ik} \) are factor loadings. The quantities \( u_i \) (individual factors) are statistically independent from the common factors.

Keywords: stochastic processes; factor analysis; cluster analysis; maximum likelihood method.

2010 Mathematics Subject Classification: 60G05; 60G35; 62H25; 93E12; 94F12.
If communalities are about 1 (i.e., the individual fluctuations are sufficiently small), one can omit the individual corrections in Eq. (102) and obtain the following approximate representation

$$x_i \approx m_i + \sum_{k=1}^{p} a_{ik} f_k.$$  (103)

Eq. (103) means that the vectors \((x_1, x_2, \ldots, x_n)\) are contained in a \(p\) dimensional domain (linear manifold) of \(n\) dimensional linear vector space. The reduction of dimensionality of the space segment filled by the stochastic process is obviously due to correlation between data for different time moments.

### Clustering time series

When operating with time series it is important to know if the current stochastic process represented by experimental data belongs to a specific process class. Let us assume that the following characteristics of the class are available: means and rms deviations (in fact, their statistical estimates) and factor loadings of the time series. We suggest to solve the problem of class membership using the test of significance algorithm and taking into consideration two reasons. First, the rms deviation of the observed values with respect to their approximation through common factors should not be large if the process belongs to the given class and factor analysis has shown a considerable contribution of common factors to correlation between terms of the time series. Second, the magnitudes of common factors minimizing the rms deviations should not considerably exceed unity because common factors are normalized random variables.

### Concluding remarks

The approach to stochastic processes proposed in this work and based on factor analysis has the advantage that it can be used both for stationary and non-stationary time series and allows one avoiding time-frequency transformation. It is in contrast with the Fourier transform method that information on the process class as a whole is contained in the factor loadings whereas information on a specific realization of the process is related to common factors which are normalized independent random variables. This makes possible mapping individual features of a process realization into a linear manifold with dimensionality which is equal to the number of common factors and thus it may be much less than dimensionality of the space spanned by experimental data. The method suggested allows performing discriminant analysis of random processes. The example given in this paper proves its efficiency. Statistical simulation of stochastic processes with preset characteristics could be one more application of the method.

### References


Applications of predictive control to opinion dynamics models on time scales

Ricardo Almeida$^1$, Ewa Girejko$^2$, Luís Machado$^3$, Agnieszka B. Malinowska$^4$, Natália Martins$^5$

1, 5 University of Aveiro, Portugal 2, 4 Białystok University of Technology, Poland

E-mail: $^1$ricardo.almeida@ua.pt; $^2$e.girejko@pb.edu.pl; $^3$lmiguel@utad.pt; $^4$a.malinowska@pb.edu.pl; $^5$natalia@ua.pt

Abstract The aim of this talk is to analyse the behaviour of opinion dynamics models on time scales, by introducing a control parameter. We observe that the predictive control method is an efficient tool for increasing the consensus speed.

Introduction

In the last decades, consensus theory has gained attention within the scientific community of different areas, like robotics, sociology, economics, political sciences, sociophysics, biology, just to name a few. In networks of individuals (called agents or experts), we say that they reach a consensus if there exists an agreement upon certain quantities of interest, that depends on the state of all agents. The consensus theory seeks conditions under which the states of all agents converge asymptotically toward a common value.

The theory of time scales, established in 1988 by S. Hilger and B. Aulbach (see [1]), is recognized as an important tool for the unification and extension of existing results for continuous and discrete-time dynamical models to nonhomogeneous or hybrid time domains. In this talk we deal with an extension of the consensus model presented in [2] on an arbitrary time scale with a predictive control.

Main results

Let $T$ denotes a time scale. In what follows we consider that $0 \in T$ and $T \in T$ is a positive real number such that $\rho(T) > 0$, $x \in C^1_{prd}$ and $u \in C_{prd}$. Our new Krause’s type model is the following:

$$
\begin{cases}
    x_i^N(t) = \sum_{j=1}^{N} a_{ij}(t)(x_j(t) - x_i(t)) + u_i(t), & t \in [0, T], \ i = 1,\ldots, N \\
    x_i(0) = x_i, & i = 1,\ldots, N
\end{cases}
$$

(104)

where the coefficients $(a_{ij})$ are given by

$$
a_{ij}(t) = \begin{cases}
    \frac{1}{\Sigma_{l=1}^{N} |x_l(t) - x_i(t)|} & \text{if } |x_j(t) - x_i(t)| < 1 \\
    0 & \text{if } |x_j(t) - x_i(t)| \geq 1
\end{cases}
$$

Keywords: Consensus formation, opinion dynamics, time scale theory.

2010 Mathematics Subject Classification: 39A12; 34N05.
Let $A$ be the matrix whose entries are $(a_{ij})$ and denote by $L$ the Laplacian of $A$. Let $x = (x_1, \ldots, x_N)$ and $u = (u_1, \ldots, u_N)$. Then, system (104) can be rewritten in matrix form as

$$
\begin{aligned}
\dot{x}(t) &= -Lx(t) + u(t), \quad t \in [0, T]
\end{aligned}
$$

subject to (104). Let $(x, u)$ be a weak local minimum for problem (104). We assume that the following assumption is met:

$$
(H) \quad I - \mu(t)L \text{ is nonsingular, for all } t \in [0, \rho(T)].
$$

Using the Weak Maximum Principle on time scales (3) we can prove the following result, where $\|x\|_C := \max_{t \in [0, T]} |x(t)|$.

**Theorem 31.** Assume that hypothesis $(H)$ is verified, and let $(x, u)$ be a weak local minimum for problem (104). Then, there exists a unique function $\overline{p}: [0, T] \rightarrow \mathbb{R}^N$, $\overline{p} \in C^1_{prd}$, such that $\|\overline{p}\|_C \neq 0$ and satisfying the following conditions:

1. **the adjoint equation:** for all $t \in [0, \rho(T)]$, and $i = 1, \ldots, N$,

$$
\overline{p}_i(t) = \sum_{j=1}^{N} \left[ 2(x_j(t) - x_i(t)) + a_{ij}(t)\overline{p}_j(t) - a_{ji}(t)\overline{p}_i(t) \right];
$$

2. **the stationary condition:** for all $t \in [0, \rho(T)]$,

$$
\overline{p}(t) = -u(t);
$$

3. **the transversality condition:**

$$
\overline{p}(T) = 0.
$$

We propose a predictive control algorithm for system (105) and provide numerical simulations showing its efficiency.

**References**


High-order Numerical Scheme for Abel’s and Generalized Abel’s Integral Equations

Kamlesh Kumar, Rajesh K. Pandey

Department of Mathematical Sciences, Indian Institute of Technology (BHU) Varanasi, UP India

E-mail: kkp.iitbhu@gmail.com, rkpandey.mat@iitbhu.ac.in

Abstract In this article, we discuss numerical scheme for two type of problem namely problem of Type I and Type II. Two numerical schemes are formulated for each of problem and analysed for solving the first kind Abel’s integral equation (Type I) and generalized Abel’s integral equation (Type II). The Numerical schemes of problem of Type I and Type II are based on the mainly cubic order polynomial approximation. The rate of convergence are derived and results of numerical experiments are presented which support the theoretical results. Numerical examples are considered from literature to perform the numerical investigations and the obtained numerical results are discussed.

References


Solution of the Front Type in the Reaction-Diffusion System with Periodic Conditions

Alina A. Melnikova\(^1\), Natalia T. Levashova\(^2\)

Moscow Lomonosov State University, Russian Federation

E-mail: \(^1\)melnikova@physics.msu.ru; \(^2\)natasha@npanalytica.ru

Abstract We consider a system of singularly perturbed equations of the reaction-diffusion type. The conditions under which the system has a solution in the form of a moving front are determined, and the existence theorem for the solution is proved. In the proof, we use the method of differential inequalities.

Introduction

Consider the initial-boundary value problem

\[
\begin{align*}
\varepsilon & (u_{xx} - u_t) = f(u, v, x, t, \varepsilon), \quad \varepsilon^2 (v_{xx} - v_t) = g(u, v, x, t, \varepsilon), \quad x \in (0, 1), \ t \in \mathbb{R}^+; \\
 u_x(0, \varepsilon, t) &= u_x(1, t, \varepsilon) = 0, \quad v_x(0, t, \varepsilon) = v_x(1, t, \varepsilon) = 0, \quad t \in \mathbb{R}^+; \\
 u(x, t, \varepsilon) &= u(x, t + T, \varepsilon), \quad v(x, t, \varepsilon) = v(x, t + T, \varepsilon), \quad x \in [0, 1], \ t \in \mathbb{R}^+.
\end{align*}
\]

(106)

where \( \varepsilon > 0 \) is a small parameter, \( f(u, v, x, t, \varepsilon) \) and \( g(u, v, x, t, \varepsilon) \) are sufficiently smooth and \( T \)-periodic in \( t \) functions. We investigate the question of the existence of a periodic solution of a system of equations (106) in the form of a moving front.

Main results

The system (106) is a reaction-diffusion type and is used in applications for modeling autowave processes of various nature. We prove the existence theorem for a periodic solution in the form of a moving front and describe the method of constructing the asymptotic approximation of the solution. We use the theory of contrast structures to obtain the asymptotics [1, 2]. We apply the method of differential inequalities [3, 4] for the proof of the existence theorem. We consider the following hypotheses:

(H\(_1\)) The equation \( f(u, v, x, t, 0) = 0 \) treated as an equation for \( u \) has exactly three roots \( u = \varphi^i(v, x, t) \in I_u, \ i = 1, 2, 3, \) such that \( \varphi^1(v, x, t) < \varphi^2(v, x, t) < \varphi^3(v, x, t) \) everywhere in the domain \( (v, x, t) \in I_v \times [0; 1] \times \mathbb{R}^+; \) moreover \( f_u(\varphi^1(v, x, t), v, x, t, 0) > 0, \) and \( f_u(\varphi^2(v, x, t), v, x, t, 0) < 0. \) (Here \( I_u \) and \( I_v \) are some ranges of variables \( u \) and \( v. \))

(H\(_2\)) Each of the equations \( h^i(v, x, t) := g(\varphi^i(v, x, t), v, x, t, 0) = 0, \ i = 1, 3 \) has a unique solution \( v = \varphi^i(x, t) \in I_v; \) moreover, the inequalities \( \varphi^1(x, t) < \varphi^3(x, t) \) and \( h_{v}^i(\varphi^i(x, t), x, t) > 0, \ i = 1, 3 \) hold on the entire domain \( (x, t) : [0; 1] \times \mathbb{R}^+). \)

Keywords: singular perturbation; reaction-diffusion; transition layer; moving front; periodic solution.

2010 Mathematics Subject Classification: 26A33; 34A60; 34G25; 93B05.
The inequalities $f_v(u,v,x,t,0) < 0$ and $g_u(u,v,x,t,0) < 0$ hold everywhere in the domain $\{(u,v,x,t) \in I_u \times I_v \times [0;1] \times \mathbb{R}^+\}$.

There exists a unique $T$-periodic solution $v_0(t), x_0(t)$ of the system of equations

$$
\begin{align*}
Jv(x,v) &:= \int y_1(x,t)h_1(v',x,t) dv' + \int y_3(x,t)h_3(v',x,t) dv' = 0, \\
Ju(x,v) &:= \int y_1(x,t) f(u,v,x,t,0) du = 0,
\end{align*}
$$

(107)

defined in the domain $\{x(t) \in (0;1), v(t) \in \{v_1(x,t), v_3(x,t)\}, (x,t) \in [0;1] \times \mathbb{R}^+\}$.

The Jacobian of the system (107) satisfies inequality

$$
\frac{D(Jv,Ju)}{D(v,x)}(v_0(t),x_0(t)) < 0, \quad t \in \mathbb{R}^+.
$$

The upper and lower solutions of problem (106) are constructed on the basis of the asymptotic approximation of the n-th order in the small parameter. According to the method of differential inequalities [3, 4] the existence of an upper and lower solution proves the existence of an exact solution of problem (106).

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**References**


Neural network method of restoring an initial profile of the shock wave

Dmitriy A. Tarkhov¹, Tatiana A. Shemyakina², Alexandra R. Beliaeva³, Ildar U. Zulkarnay⁴

¹,²,³ Peter the Great St. Petersburg Polytechnic University, Russia
⁴ Bashkir State University, Russia

E-mail: ¹dtarkhov@gmail.com ²sh_tat@mail.ru ³alex.belyaeva2012@gmail.com ⁴zulkar@mail.ru

Abstract  Neural network modeling is applied to the solution of the inverse problem of mathematical physics. The initial pressure distribution profile in the pipe, in which a shock wave is propagated, restored from the measurement results of the sensor.

Introduction  Recovery process and identifying the causes of the accident or disaster is one of the important problems of technogenic safety. Blast waves are one of the objects of study such processes. The consequence of an explosion could be the formation of surface discontinuities currents, which are referred to as shock waves. In this work neural network modeling is considered in order to clarify the processes of occurrence and propagation of the shock wave. The object of study is the primary incident shock wave in an atmospheric shock tube. The pressure distribution profile at the initial time is based on the use of differential equations and experimental data.

Experiment and results  Classic experiment of occurrence and distribution of the shock wave is considered in this work. The cylindrical tube with a closed inlet and outlet is divided by diaphragm into two chambers: the left one is a high-pressure chamber (HPC), and the right one is the low-pressure chamber (LPC). The tube is thermally insulated and the gas motion is adiabatic. The driver gas is in the high-pressure chamber. The atmospheric air is used as the driver gas and the pressure drop is created by filling the sealed LPC with the working gas the pressure of which is less than atmospheric pressure. The experiments were conducted at the initial pressure of 30, 60, 90 mm Hg in LPC. The diaphragm is destroyed at some time. After the rupture of the diaphragm the driver gas rushes from the high-pressure chamber to the low-pressure chamber, forming a compression wave which forms a shock wave. The pressure profiles were recorded by sensors in the LPC shock tube. These data were transformed using the program "L-Graf"in the dependence of pressure on time that elapses while the shock wave passes through the LPC. Unsteady flows of gas and spread of disturbance is observed in the shock tube after the rupture of the diaphragm. This process is described by a system of two first order linear partial differential equations with the initial conditions.

The neural network model is built to restore the initial pressure profile. The model uses the data from the pressure sensors along the tube. The output of the neural network model at the initial time gives the initial distribution. The approximate solution is constructed in the form of an artificial neural network by known methods in the form of perceptrons with one hidden layer. The problems where the solution or the approximation is spasmodic, it is better to use a neural network with sigmoid-type basis functions. This function is a hyperbolic tangent. Computational experiments conducted for this task confirmed the correctness of this assumption.

Keywords: neural networks, nonlinear optimization, shock wave, differential equations, inverse problem.

2010 Mathematics Subject Classification: 62M45
The adjustment of linear and nonlinear parameters was carried out by minimizing the functional, which takes into account the error in satisfying each of the equations, and the differences in the values of network output for \( v; q \) from the corresponding experimental data.

The approximate neural network solution [1–3] was built for each value of the initial pressure drop for different numbers of neurons \( n \). During the research, we obtained the following results. A sharper pressure drop in LPC is obtained with increasing initial pressure.

The accuracy of approximation falls in the vicinity of the wave. This is explained as follows. The approximating function has a discontinuity of the first derivative. The function corresponding to the neural network is smooth. By assumption, different time profiles of pressure on the sensor correspond to different initial spatial profiles of the pressure, resulting from the rupture of the diaphragm. Our method allows restoring the data profiles. But it is difficult to verify the result experimentally. Indirectly, this can be verified, if we calculate the profile of the reflected wave using a neural network model and compare it with the experimental data. This will be the subject of additional research.

Conclusion

Neural network modeling gives a 3D model of the shock wave. The model represents the pressure dependence of the coordinate and time. The results of the calculations have shown the ability to restore the initial pressure distribution along the tube by means of measuring the dependence of the pressure on the time indices on the sensor.

Further supplementary data used in the training of the neural network will be provided. For example, we can add an asymptotic solution. This should significantly improve the accuracy and reduce training time.

References


Some properties of univalent holomorphic functions by using subordination

Shahram Najafzadeh
Payame Noor University, Tehran, Iran
E-mail: najafzadeh1234@yahoo.ie

Abstract The purpose of the Present paper is to Investigate some Inclusion Properties of Certain Subclasses of Analytic Functions Associated with a Family of Multiplier Transformations on the class \( A \) of analytic Functions in the unit disc \( U = \{ z : |z| < 1 \} \).

References


Keywords: Convolution property; analytic function; multiplier transformation; subordination; starlike function; convex function.
2010 Mathematics Subject Classification: 30C45; 30C50.
Synthesis of Adequate Mathematical Description for Dynamical Systems

Yuri L. Menshikov
Dnepro University, Ukraine

E-mail: Menshikov2003@list.ru

Abstract The problem of synthesis of adequate mathematical description for dynamical systems is considered. This problem is reduced to solution of some inverse problems with features. These features lead to change of method solution and interpretation of solutions.

Introduction

Let us consider the dynamical system the motion of which is described by system of ordinary linear equations

\[ \dot{x} = Ax + Bz, x(t_0) = x^0. \]  

(108)

where \( x(t) = (x_1(t), ..., x_n(t))^T \in X \) is vector function of state variables, \( z(t) = (z_1(t), ..., z_m(t))^T \in Z \) is vector function of external load, \((\cdot)^T\) is the sign of transposition; \( A, B \) are matrices corresponding orders; \( Z, X \) are the normal functional spaces. Let us assumed for simplicity that only one variable of state \( x_j(t) \) has interests with mathematical modelling of physical processes point of view. Also we will assume that vector function of external load \( z(t) = (z_1(t), ..., z_m(t))^T \) has only one nonzero component \( z_k(t) \). By a mathematical description of a dynamic process, we mean the system of equations (108), the external load function \( z(t) \) and the initial conditions in the aggregate.

Definition. The mathematical description (108) will be called adequate with respect to a variable \( x_j(t) \) with accuracy \( \delta \) if the results of mathematical modeling of the state variable \( x_j(t) \) coincide with the experimentally obtained function \( \tilde{x}_j(t) \) with accuracy \( \delta \):

\[ \| \tilde{x}_j(t) - x_j(t) \|_X \leq \delta. \]  

(109)

If the error value \( \delta \) is equal to the experimental error, then the estimate (109) will be objective. If the mathematical description (108) is not adequate on the variable \( x_j(t) \), then the results of mathematical modeling of the function \( x_j(t) \) using this description will not be trustworthy. For system (108), several variants of problems can be considered for the purpose of constructing an adequate mathematical description: matrices \( A, B \) are given, an experimental function \( \tilde{x}_j(t) \) is given, initial conditions are given, and it is required to determine the function \( z_k(t) \) when condition (109) is satisfied. Changing the initial conditions, matrices \( A, B \) and function \( z_k(t) \) by places, one can formulate a number of problems for the purpose of constructing an adequate mathematical description. Such problems are used in problems of motion control, in problems of optimizing the parameters of motion of a dynamic system, in forecasting problems. They can be combined into one inverse problem: to construct an adequate mathematical description of the physical process from experimental data \( \tilde{x}_j(t) \). This problem can be written in the form

\[ \tilde{A}z = u_\delta. \]  

(110)

Keywords: dynamical systems; adequate mathematical description; synthesis; ill-posed problems; features.

2010 Mathematics Subject Classification: 65L06; 65J20.
where $\tilde{A}$ is compact operator; $\tilde{A}: (Z \to U), z \in Z, u_\delta \in U, u_\delta$ is initial data, $z$ is unknown solution; $Z, U$ are normal functional spaces. In what follows we assume that $u_\delta$ in equation (110) is given known inaccuracy $\delta$ [1]:

$$\|u_\delta - u_\delta\| \leq \delta_0,$$

where $u_\delta$ is exact initial data.

**Main results**

However, this inverse problem has a number of specific features [2]: in these problems it makes no sense to take into account the error of the operator $\tilde{A}$, since in the future mathematical modeling (simulation) will use an approximate mathematical description (108); it also makes no sense to require the approximate solution $z_\delta$ of equation (110) to converge to the exact solution of (110) $z_\delta = u_\delta$ for $\delta \to 0$, since the operator $\tilde{A}$ is an approximate one. Therefore, when constructing an adequate mathematical description for the purposes of mathematical modeling, we can abandon the assumption that the exact solution of equation (110) $z_\delta$ belongs to the set of possible solutions $Q_\delta = z : \|\tilde{A}z - u_\delta\| \leq \delta$. This set is unbounded because of the compactness of the operator $\tilde{A}$. For the purposes of mathematical modeling, any element of the set $Q_\delta$ is suitable. In view of this, additional requirements can be imposed to obtain a unique approximate solution (110), for example, the convenience of solution for purposes of mathematical modeling (the least number of Fourier coefficients in the expansion, the extreme value a priori of a given functional $\Omega[z]$, etc.). We consider the extremal problem

$$\Omega[z_\delta] = \inf_{z \in Q_\delta} \Omega[z].$$

**Theorem 32.** Suppose $Z$ is a functional Hilbert space, $\Omega[z]$ is highly convex functional and semicontinuous from low on $Q_\delta$, Lebesgue set for any constant is a compact in $Z$. Than the solution of (5) exists, unique, belongs to $Q_\delta$ and it stable to small changes in the initial data $u_\delta$.

In this case, Tikhonov’s regularization algorithm can also be used to find the approximate solution (110). The regularization parameter will then correspond to the Lagrange multiplier and the discrepancy method can be used to find a solution satisfying the equality (109).

**References**


On the Criteria for the Critical State of a Round Rod exposed to Tension

Valery Dilman¹, Tatyana Karpeta²

¹South Ural State University, Russia ²South Ural State University, Russia

E-mail: ¹dilman49@mail.ru; ²etv@mail.ru

Abstract Under investigation are the mathematical models of stressed states of cylindrical rods with a less strong heterogeneous transverse insert under axial load. On the basis of the analytical method developed by the authors, the force criterion of the critical state for such joints was obtained, which makes it possible to numerically determine the critical axial loads for mechanically heterogeneous welded joints of reinforcement rods. The internal boundary-value problem (the conjunction problem) for stresses on the contact surface has been solved. It is shown that even very thin interlayers reduce the strength of the joint.

Introduction

In this paper, mathematical models of stressed states of cylindrical rods with a less strong heterogeneous transverse insert under axial load are under consideration. The hypothesis of flat cross sections \( v_z = W(z) \) is assumed to be made use of (\( v_z \) is the strain rates in the direction of the cylinder axis). In this case, the axisymmetric stress-strain state of a plastic body is determined with a system of equations:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} = 0; \quad \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} = -\frac{\tau_{rz}}{z},
\]

\[
\sigma_z - \sigma_r = \pm 3\sqrt{(\sigma_{cr})^2 - \tau_{rz}^2},
\]

\[
\frac{3W'(z)}{\sigma_z - \sigma_r} = -\frac{rW''(z)}{2\tau_{rz}}.
\]

Here \( \sigma_z, \sigma_r \) and \( \tau_{rz} \) are normal and tangential stresses, respectively. System (113)–(115) has a hyperbolic type that allows us to use the method of characteristics when studying it. The domain of the scope of the unknown functions is the area of the axial section of the insert, i.e. a rectangle \([-1; 1] \times [-\kappa; \kappa]\). Obvious conditions are satisfied: \( \sigma_r(1, z) = 0; \ \tau_{rz}(1, z) = 0; \ \tau_{rz}(0, z) = 0; \ \tau_{rz}(r, 0) = 0 \). At the contact boundary (between the main material of the rod and the insert), the conditions are satisfied:

\[
\sigma_z^+ = \sigma_z^-, \ \tau_{rz}^+ = \tau_{rz}^-.
\]

The bearing capacity of the joint, as the average ultimate stress \( \sigma_{cr} \), is calculated from the formula

\[
\sigma_{cr} = 2r \int_0^1 \sigma_z(r, \kappa) \, dr.
\]
We introduce the following notation: $K_l = \sigma^{+\epsilon}(\varkappa)/\sigma^{+\epsilon}(0)$, $K = \sigma^{+\epsilon}(\varkappa)/\sigma^{-\epsilon}(0)$, where $\sigma^{+\epsilon}$ and $\sigma^{-\epsilon}$ are the ultimate strengths of the main material and the material of the insert.

Under the action of a tensile load, the increase of contact hardening in a less solid layer occurs until either (the first case of a critical state) the base metal goes over to the plastic state in the proximity to the contact surface or (the second critical state case) until the tangential stresses reach their maximum value, which, as follows from (114), is equal to $\sigma^{\epsilon\epsilon}$. Therefore, the first case of the critical state is determined by the inequality at the contact boundary $\max_{z=\varkappa} \tau_{rz}(r,z) < \sigma^{\epsilon\epsilon}$.

**Main results**

Solving the conjunction problem arising from conditions (116) for stresses on the contact surface [1], it is established that there exists such a segment of contact boundary $FA$ that $\max_{z=\varkappa} \tau_{rz}(r,z) = \tau_{rz}(A) = \tau_{rz}(F)$, where $A$ is the exit point of the contact surface to the free one. Wherein:

1) the coordinate of point $F$:

$$r_F = 1 - 1,995 \varkappa - 0,017 \varkappa^2 + (0,32 \varkappa + 0,283 \varkappa^2)(K - 1);$$

(118)

2) the dependence of tangential stresses in section $FA$ of the contact boundary (at the critical moment of loading) on the parameters of the layer; in particular, with $\varkappa < 0,5$ at point $F$ this dependence takes the following form:

$$\tau_{rz}(F) = (K_f(K - 1)(0,8 + 0,707 \varkappa) + 0,013 - 0,043 \varkappa)\sigma^{\epsilon\epsilon}(0);$$

(119)

3) the dependence of the normal stresses $\sigma_z$ in section $FA$ on the contact boundary (at the critical moment of loading) on the parameters of the layer; in particular with $\varkappa < 0,5$

$$\sigma_z(F) = K_f(1 + (0,487 - 0,008 \varkappa - 0,035 \varkappa^2)(K - 1) - (0,169 + 0,049 \varkappa + 1,066 \varkappa^2)(K - 1)^2)\sigma^{\epsilon\epsilon}(0).$$

(120)

The increase of the contact hardening of the insert takes place until the main material in the vicinity of the contact surface goes into a plastic state which is determined by condition

$$\max_{z=\varkappa} \tau_{rz}(r,z) < \sigma^{\epsilon\epsilon}.$$  

(121)

At this point, the joint reaches a critical state. In view of (119) and (121), we obtain the condition under which the base metal is involved in plastic deformation: that is the criterion of the critical state of the joint. Formulas (118)–(120), together with the results of [2], allow us to calculate, with the use of software tools, the average critical stress according to formula (117) as a function of the values of parameters $K$, $K_f$ and $\varkappa$.

**References**


Numerical Scheme for Generalized Fractional Integro–Differential Equations

Rajesh K. Pandey¹, Kamlesh Kumar²

Department of Mathematical Sciences, Indian Institute of Technology (BHU) Varanasi, India

E-mail: ¹rkpandey.mat@iitbhu.ac.in; ²kkp.iitbhu@gmail.com

Abstract The aim of this paper is to develop the numerical schemes for the generalized fractional integro-differential equations defined in terms of the B-operators introduced recently. The schemes are based on the approximation of the unknown function in terms of the linear and quadratic polynomials. The convergence of the schemes are also established. Illustrative examples with different kernels in B-operators are considered to perform numerical investigations. The obtained results are presented. As B-operators reduces Caputo, Riesz- Caputo and many other derivatives defined in literature therefore the schemes presented here could be applied to many other fractional integro-differential equations defined using these fractional derivatives.

References


A matrix approach to the representation of orthogonal hypercomplex polynomial systems

Isabel Cacao\textsuperscript{1}, Helmuth Malonek\textsuperscript{1}, Graca Tomaz\textsuperscript{2}

\textsuperscript{1}University of Aveiro, Portugal \quad \textsuperscript{2}Polythechnical Institute of Guarda, Portugal

E-mail: \textsuperscript{1}isabel.cacao@ua.pt; hrmalon@ua.pt \textsuperscript{2}gtomaz@ipg.pt

Abstract The hypercomplex function theory generalizes the theory of holomorphic functions of one complex variable to higher dimensions. In this context, we provide a unified approach to the matrix representation of systems of orthogonal polynomials of a hypercomplex variable with values in a Clifford algebra. The core of the proposed approach is the so-called shifted generalized Pascal matrix that relates the both well-known special subdiagonal creation and shift matrices.

Acknowledgments

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References


Keywords: hypercomplex function theory; generalized Appell polynomials; matrix representation; creation matrix; Pascal matrix

2010 Mathematics Subject Classification: 30G35; 65F60; 11B83.
Collocation-Variation Difference Schemes for Differential-Algebraic Equations

Mikhail V. Bulatov¹, Liubov S. Solovarova²

¹,² Matrosov Institute for System Dynamics and Control Theory of SB RAS, Russia

E-mail: ¹mvbul@icc.ru; ²soleilu@mail.ru

Abstract We consider a particular case of constructing collocation-variation difference schemes of the first order for initial-value problems in differential-algebraic equations. The collocation-variation approach can be used for a wider class of the problems. It is possible to propose the higher order difference schemes. The schemes essentially differ from the known methods and are easy for programming.

Introduction

Consider the system of linear differential equations

\[ A(t)\dot{x}(t) + B(t)x(t) = f(t), \quad t \in [0, 1], \] (122)

\[ x(0) = x_0, \] (123)

where \( A(t), B(t) \) are \((n \times n)\)-matrices, \( f(t) \) and \( x(t) \) are the given and unknown \( n \)-dimensional vector-functions, respectively.

If \( \det A(t) \equiv 0 \), (124)

then systems of the form (122) are called differential-algebraic equations (DAE). It is assumed that the initial condition (123) is given in such a way that the problem under consideration has a solution. The general solution has the form

\[ x(t, c) = \Phi(t)c + \int_0^t K_0(t, \tau)f(\tau)d\tau + \sum_{j=1}^r K_j(t)f^{(j-1)}(t), \] (125)

where \( \Phi(t), K_0(t, \tau), K_j(t) \) are \((n \times n)\)-matrices, \( \text{rank} \Phi(t) = \text{const} \forall t \in [0, 1] \), and \( K_j(t) \neq 0 \). The integer value \( r \) is called the index of the DAE. (122) [1].

It is known (see, for example [2]) that constructing numerical algorithms for DAEs is a difficult problem.

Keywords: differential-algebraic equations; index; difference schemes.

2010 Mathematics Subject Classification: 65L80.
Main results

We propose collocation-variation difference schemes for problem [122], [123]. For simplicity, we consider only the interpolation variant of the difference schemes with one (right) collocation point. Let $L_2(x_{i+2}, x_{i+1}, x_i, t)$, $t \in [t_i, t_{i+2}]$ be a vector-polynomial passing through points $(x_{i+2}, t_{i+2}), (x_{i+1}, t_{i+1}), (x_i, t_i)$. By substituting $L_2(.)$ into [122] and setting $t = t_{i+2}$, we obtain the well-known two-step scheme

$$A(t_{i+2})(3x_{i+2} - 4x_{i+1} + x_i) + 2hB(t_{i+2})x_{i+2} = 2hf_{i+2}. \tag{126}$$

We regard this scheme as a one-step method and an equality constraint, to which we add a condition for minimum

$$\|L_2(.)\|^2 = \sum_{j=0}^2 \int_{t_i}^{t_{i+2}} L_2^{(j)}(x_{i+2}, x_{i+1}, x_i, t) L_2^{(j)}(x_{i+2}, x_{i+1}, x_i, t)dt \approx \tag{127}$$

$$\approx 2h\left(\|x_{i+2}\|^2 + \frac{\|x_{i+2} - x_i\|^2}{2h} + \frac{\|x_{i+2} - 2x_{i+1} - x_i\|^2}{h^2}\right), \quad x_0 = x(0).$$

As multiplication does not influence finding a minimum of a function, $x_{i+1}, x_{i+2}$ can be found from the solution of the quadratic programming problem

$$\Phi(x_{i+2}, x_{i+1}) = h^2\|x_{i+2}\|^2 + \frac{h^2}{4}\|x_{i+2} - x_i\|^2 + \|x_{i+2} - 2x_{i+1} - x_i\|^2 \rightarrow min \tag{128}$$

with the equality constraint [126]. Here $\|\cdot\|$ denotes the Euclidian norm.

The quadratic programming problem [128], [126] is reduced to the solution of the system of linear algebraic equations, using the method of Lagrange multipliers.

Construction of collocation-variation difference schemes is based on an idea from [3], [4]. The analysis of the scheme and the numerical calculations of the test examples are given.

Acknowledgments

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References


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The approximate solution of nonlinear Volterra Hammerstein integral equations with RH wavelet bases in a complex plane

Majid Erfanian¹, Abas Akrami², Hamed Zeidabadi³

¹University of Zabol, Iran  ²University of Zabol, Iran  ³University of Damghan, Iran

E-mail: ¹erfaniyan@uoz.ac.ir; ²akramiabas@yahoo.com; ³h.zeidabadi@yahoo.com;

Abstract  In this article we calculated the solutions of nonlinear Volterra Hammerstein integral equations of second kind, by uses the properties of rationalized Haar wavelets. The main tool for error analysis is using the Banach fixed point theorem and we get an upper bound for the error. Furthermore, the order of convergence is analyzed. The algorithm to compute the solutions and some numerical examples are also illustrated. The numerical results obtained by the our method have been compared with other methods.

References


Keywords  Nonlinear Volterra Hammerstein integral equation; Rationalized Haar wavelet; Operational matrix; fixed point theorem; complex plane.

2010 Mathematics Subject Classification  47A56; 45B05; 47H10; 42C40.
On Some Properties of Higher Order Differential Algebraic Equations Perturbed by the Fredholm Operator

Elena Chistyakova\textsuperscript{1}, Victor Chistyakov\textsuperscript{2}

\textsuperscript{1,2} Matrosov Institute for System Dynamics and Control Theory of SB RAS, Russia

E-mail: \textsuperscript{1}elena.chistyakova@icc.ru; \textsuperscript{2}chist@icc.ru

Abstract We consider higher order differential algebraic equations perturbed by the Fredholm operator. We introduce the notion of index for such problems and propose a numerical method of solution based on the least squares method.

Introduction

We consider systems of higher order ordinary differential equations of the form

$$\Lambda_k x := A_k(t)x^{(k)}(t) + A_{k-1}(t)x^{(k-1)}(t) + \ldots + A_0(t)x(t) = f(t),$$

(129)

where $t \in T := [\alpha, \beta]$, $A_i(t)$ are $n \times n$-matrices, $i = 1, k$, $x(t)$ and $f(t)$ are the desired and the known vector-functions, correspondingly, $x^{(j)}(t) = (d/dt)^j x(t)$, with the initial data

$$x^{(j)}(a) = a_j, j = 0, k - 1,$$

(130)

where $a_j$ are vectors from $\mathbb{R}^n$. It is assumed that the entries of (129) are smooth enough and the following condition holds

$$\det A_k(t) = 0 \forall t \in T.$$ 

(131)

DAEs arise in the mathematical modeling of a wide variety of problems from engineering and science such as in multibody and flexible body mechanics, electrical circuit design, optimal control, incompressible fluids, molecular dynamics, chemical kinetics, and chemical process control. If the process under study has an aftereffect, then DAEs might include Volterra and Fredholm equations. In this talk, along with system (129), we consider a system

$$(\Lambda_k + \lambda \Phi) z = f, \quad \Phi y = \int_{\alpha}^{\beta} K(t, s) z(s) ds,$$

(132)

where $\lambda$ is some parameter, $K(t, s)$ is an $n \times n$-matrix defined in $T \times T$, $z(t)$ is the desired vector-function, with the initial data

$$z^{(j)}(a) = a_j, j = 0, k - 1.$$

(133)

Systems (129) with the condition (131) are commonly referred to as differential algebraic equations (DAEs), the term was first introduced in [1]. By present time, DAEs of the first order (when $k = 1$ in (129)) have been fairly well studied, and, as a rule, any system (129) can be reduced to a first order system by a change of variables. However, if $k > 1$, the DAE (129) possesses a number of interesting properties, which are lost during reduction.

We investigate the connection between solutions to (129) and (132) and propose a numerical method for solving the initial problem (132), (133).

Keywords: differential algebraic equations; index; Fredholm equations; least squares method.

2010 Mathematics Subject Classification : 34A09; 65L80; 45J05.
Main results

Index is a notion used in the theory of DAEs for measuring the complexity of a given DAE. The generalized notion of index for higher order DAEs has the form:

**Definition 5.** If there exists the operator \( \Omega_l = \sum_{j=0}^{l} L_j(t)(d/d)^j \), where \( L_j(t) \) are \( n \times n \)-matrices from \( C(T) \), such that \( \Omega_l \circ \Lambda_k y = \tilde{A}_k(t)y^{(k)}(t) + \tilde{A}_{k-1}(t)y^{(k-1)}(t) + \ldots + \tilde{A}_0(t)y(t) \) \( \forall y(t) \in C^{l+k}(T) \), where \( \tilde{A}_i(t) \) are some \( n \times n \)-matrices from \( C(T) \), \( i = 0, k \), \( \det \tilde{A}_k(t) \neq 0 \) \( \forall t \in T \), then the smallest possible \( l \) is said to be the index of \( (129) \).

The index provides an important information on solvability of the DAE under study, and on the basis of Definition 5 we can prove the following theorem.

**Theorem 33.** Let the following conditions be satisfied:

1. \( A_i(t) \in C^m(T), i = 0, k, m = \max((k-1)n + r + 1, 2l) \), \( r = \max(\text{rank} A_k(t), t \in T) \);
2. system (1) is index \( l \) and \( K(t, s) \in C^l(T \times T), f(t) \in C^l(T) \) in system (4).

Then, system (4) is solvable for all \( \lambda \), except maybe a countable set \( \{ \lambda_i, i = 0, 1, 2, \ldots \} \), and its general solution at \( \lambda \neq \lambda_i \) has the form

\[
z(t, c) = Z_d(t)c + g(t), \quad t \in T
\]

where \( Z_d(t) = (I_n + \lambda W)X_d(t), \ g(t) = (I_n + \lambda W)\psi(t), \ W \) is some Fredholm operator, \( X_d(t) \) is an \( n \times d \)-matrix from \( C^k(T), c \) is an arbitrary constant vector, \( \psi(t) \) is some vector-function from \( C^k(T), I_n \) is an identity matrix of dimension \( n \).

Additionally, the linear combination \( X_d(t)c + \psi(t) \) is a solution to \( (129) \) for any \( c \).

The numerical solution of \( (132), (133) \) is based on the least squares method, which convergence was proven and verified by a number of illustrative test examples. The current study continues investigations started in [2].

**Acknowledgments**

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**References**


Modelling of incompressible fluid by means of partial differential algebraic equations

Victor Chistyakov

Matrosov Institute for System Dynamics and Control Theory of SB RAS, Russia

E-mail: chist@icc.ru

Abstract We consider linear partial differential algebraic equations with constant coefficients and focus on the evolutionary systems. We introduce the notion of index for partial differential algebraic equations and study incompressible fluid models described by this class of equations.

Introduction

We consider incompressible fluid models written in the form of a system of partial differential equations

\[ \Lambda_k(D_t, D_x) u := \sum_{j=0}^k A_j(D_x) D_t^j u = f(x, t), \quad (x, t) \in U = \mathbb{R}^m \times [t_0, t_1], \]  

where \( A_j(D_x) \) are \( n \times n \)-matrices which elements are differential operators with respect to \( x \) with constant coefficients, \( D_x \equiv (\partial/\partial x_1, \partial/\partial x_2, \ldots, \partial/\partial x_m) \), \( D_t \equiv \partial/\partial t \); \( f(x, t), u \equiv u(x, t) \) are the given and the desired vector-functions, correspondingly. Introduce the matrices \( A_j(\mu), \sum_{j=0}^k A_j(\mu) \lambda^j \), where \( \mu = (\mu_1, \mu_2, \ldots, \mu_m) \) and \( \lambda \) are some parameters (generally, complex ones: \( \mu \in \mathbb{C}^m \), \( \lambda \in \mathbb{C} \)). Following [1], we call such matrices symbols of the operators \( \Lambda_j(D_x), \Lambda_k(D_t, D_x) \). It is assumed that

\[ \det A_k(\mu) = 0 \quad \forall \mu \in \mathbb{C}^m, \]  

and that \( f(x, t) \) is sufficiently smooth in \( U \). By the solution of (134) we understand a vector-function \( u \) that has continuous partial derivatives with respect to \( x \) and \( t \) of the required order and satisfies \( \Lambda(D_t, D_x) u \equiv f(x, t), \quad (x, t) \in U \). Symbols of operators are said to be regular, if

\[ \det A_j(\mu) \neq 0, \quad \det \sum_{j=0}^k A_j(\mu) \lambda^j \neq 0, \quad \mu \in \mathbb{C}^m, \quad \lambda \in \mathbb{C}. \]

Systems (134) with the condition (135) are commonly referred to as partial differential algebraic equations. Beginning with the works by L.S. Sobolev, investigation of systems of the form (134) has played an important role in the theory of differential equations, which is why they are also mentioned as Sobolev type equations [2].

Keywords: differential algebraic equations; partial derivatives; Stokes problem; method of lines; index.

2010 Mathematics Subject Classification: 34A09; 35A10; 76D07.
Main results

Theorem 34. Let in system \([134]\) the symbol of the operator \(\Lambda_k(D_t, D_x)\) be regular. Then, there exists the operator

\[
\tilde{\Lambda}_l(D_t, D_x) := \sum_{j=0}^l \tilde{A}_j(D_x) D_t^j,
\]

where \(\tilde{A}_j(D_x)\) are \(n \times n\)-matrices which elements are differential operators with respect to \(x\) with constant coefficients, such that

\[
\tilde{\Lambda}_l(D_t, D_x) \circ \Lambda_k(D_t, D_x) y := \tilde{\Lambda}_k(D_t, D_x) y = \sum_{j=0}^k \tilde{A}_j(D_x) D_t^j y \quad \forall y(x, t) \in C^\infty(U),
\]

where the symbol of the operator \(\tilde{\Lambda}_k(D_x)\) is regular.

A converse statement is valid: if the symbol of the operator \(\tilde{\Lambda}_k(D_x)\) is regular, then the symbol of the operator \(\Lambda_k(D_t, D_x)\) is also regular.

The smallest possible \(l\) is said to be the index of \([134]\) with respect to \(t\). In the most primitive case, when the symbol of the operator \(\tilde{\Lambda}_k(D_x)\) is unimodal: \(\det \tilde{\Lambda}_k(u) = \text{const} \neq 0\), there exists the inverse operator \(\tilde{\Lambda}_k^{-1}(D_x) = \hat{\Lambda}_k(D_x)\), and the operator \(\hat{\Lambda}_k(D_x) \circ \tilde{\Lambda}_1(D_t, D_x)\) reduces \([134]\) to the Cauchy-Kovalevskaya form. Additionally, if in \([134]\) the symbol of the operator \(\hat{\Lambda}_k(D_x)\) is unimodal, then the solution is unique: \(u = \sum_{j=0}^\nu B_j(D_x) D_t^j f\), where \(B_j(D_x)\) are \(n \times n\)-matrices which elements are differential operators with respect to \(x\) with constant coefficients.

It was established that for some difference approximations \(D_x \approx \Delta_x\) the method of lines results in ordinary differential algebraic equations \(\sum_{j=0}^k A_{j,N} D_t^j u_N(t) = f_N(t), \quad t \in [t_0, t_1]\), where \(N\) is the grid parameter, which inherits the index of the original system. The new findings were applied to investigation of the Stokes system and the two-phase liquid filtration system.

Acknowledgments

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References


On second-order resonance periodic problem with impulses

Martina Langerová

NTIS, University of West Bohemia, Pilsen, Czech Republic

E-mail: mlanger@ntis.zcu.cz

Abstract We consider second-order nonlinear periodic problems with impulses in derivative at fixed times. We provide a sufficient existence condition for solutions of resonance problems in terms of the forcing term, restoring force and impulse functions. This condition not only generalizes the classical Landesman-Lazer condition but also implies existence results for problems with vanishing or oscillating nonlinearities. The models with solutions which include instantaneous impulses depending on the position that result in jump discontinuities in velocity, but with no change in position, are stressed in different fields of applied research.

Introduction

In this contribution, based on the joint work with P. Drábek [1], we consider the second-order periodic problem at resonance with nonlinear impulses at the derivative

\[
\begin{align*}
    x''(t) + m^2 x(t) + f(t,x(t)) &= e(t), & \text{for a.e. } t \in [0,2\pi], \\
    x(0) &= x(2\pi), \quad x'(0) = x'(2\pi), \\
    x(t_j^+) &= x(t_j^-), \quad \Delta x'(t_j) := x'(t_j^+) - x'(t_j^-) = I_j(x(t_j)), \quad j = 1,2,\ldots,p,
\end{align*}
\]

(136)

where \( m, p \in \mathbb{N}, \ f : [0,2\pi] \times \mathbb{R} \to \mathbb{R} \) is a Carathéodory function, \( e \in L^1(0,2\pi), \ 0 < t_1 < \ldots < t_p < 2\pi \) and \( I_j : \mathbb{R} \to \mathbb{R} \) is continuous, \( j = 1,2,\ldots,p \). A function \( x \in H \); \( H := \{ x \in H^1(0,2\pi) : x(0) = x(2\pi) \} \) with the scalar product \( (x,y) = \int_0^{2\pi} (x'(t)y'(t) + x(t)y(t)) \, dt \) and the norm \( \|x\| = \left( \int_0^{2\pi} (x'(t))^2 + (x(t))^2 \right)^{1/2} \). We split a function \( e \in L^1(0,2\pi) \) as \( e = \bar{e} + e^\pm \), where \( \int_0^{2\pi} e^\pm(t) \sin(mt+\theta) \, dt = 0 \) for all \( \theta \in \mathbb{R} \).

Main results

The purpose of our research was to introduce rather general sufficient condition of Landesman-Lazer type for the existence of a solution of [136] in terms of the asymptotic properties of the forcing term, restoring force and impulse functions:

If \( (x_n) \subset H \) is a sequence such that \( \|x_n\|_\infty \to \infty \) and there exists \( \theta_0 \in \mathbb{R} \) such that \( \frac{x_n}{\|x_n\|_\infty} \to \sin(mt+\theta_0) \) in \( C[0,2\pi] \), then

\[
\lim_{n \to \infty} \left( \int_0^{2\pi} \! \int_0^{x_n(t)} f(t,s) \, ds \, dt - \sum_{j=1}^p \int_0^{2\pi} I_j(s) \, ds - \int_0^{2\pi} \! \int_0^{x_n(t)} \bar{e}(t) x_n(t) \, dt \right) = \pm \infty. \quad \text{(SC)}_\pm
\]

(137)

We assume that there exist \( r \in L^1(0,2\pi) \) such that

\[
|f(t,s)| \leq r(t)
\]

Keywords: second-order periodic problem; impulsive problem; resonance problem; Landesman-Lazer condition; saddle point theorem.

2010 Mathematics Subject Classification: 34A37; 34B37; 34C25; 34F15.
for a.e. \( t \in [0, 2\pi] \) and for all \( s \in \mathbb{R} \), and a constant \( c > 0 \) such that for all \( s \in \mathbb{R} \),

\[
|I_j(s)| \leq c, \quad j = 1, 2, \ldots, p. \tag{138}
\]

Our main result is the following theorem:

**Theorem 35.** Let us assume (137), (138), and let either (SC)\(^{+}\) or (SC)\(^{-}\) hold true. Then problem (136) has at least one solution.

We apply Saddle Point Theorem due to P. Rabinowitz \(^2\) to prove this theorem.

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**References**


A description of the model of a process of the semi-Markovian walk with a delaying screen by means of a fractional order differential equation

Konul K. Omarova\textsuperscript{1}, Rovshan A. Bandaliyev\textsuperscript{2}, Aybeniz K. Omarova\textsuperscript{3}

\textsuperscript{1}Institute of Control Systems of ANAS, Azerbaijan, \textsuperscript{2}Institute of Mathematics and Mechanics of ANAS, Azerbaijan, \textsuperscript{3}Ministry of Health of the Azerbaijan Republic, Azerbaijan

E-mail: \textsuperscript{1}omarovakonulk@gmail.com; \textsuperscript{2}bandaliyev.rovshan@math.ab.az;\textsuperscript{3}konul-kamal@rambler.ru

Abstract We study a process of the semi-Markovian walk with a delaying screen given in general form by means of integral equation. In this paper, the residence time of the system is given by the gamma distribution with the parameter $\alpha = \frac{1}{2}$ resulting in a fractional order integral equation. The purpose of this paper is to reduce the fractional order integral equation to a fractional order differential equation and its solution.

References


\textbf{Keywords}: Semi-Markovian random walk process; Laplace transform; gamma distribution; fractional order differential equation.

\textbf{2010 Mathematics Subject Classification}: 60K15, 60K20, 34A08.
Stochastic Bounds for the Main Characteristics of a Single Server Queue with Two Types of Calls

Mohamed Boualem¹, Aicha Bareche²

¹,² Research Unit LaMOS (Modeling and Optimization of Systems), Faculty of Technology, University of Bejaia, 06000 Bejaia, Algeria

E-mail: ¹robertt15dz@yahoo.fr; ²aicha_bareche@yahoo.fr; ²aichabareche0912@gmail.com

Abstract In this investigation, we derive insensitive bounds for the stationary distribution of the embedded Markov chain of a single-server retrial queue with two types of calls. This model is a natural generalization of the ordinary M2/G2/1 priority queue with the head-of-the-line priority discipline and the ordinary M/G/1 retrial queue. In the case of blocking the first type calls can be queued whereas the second type calls must leave the service area but return after some random interval of time to try their luck again. The methodology is strongly based on the general theory of stochastic orders. Therefore, the obtained bounds (lower and upper) in this paper are easy to calculate and seem to be good approximations for stationary distribution of the embedded Markov chain of the considered system.

Introduction

Retrial queues have applications on numerous real-life systems such as telephone switching systems, mobile communication networks, random access protocols in wireless networks, call centers, wavelength-routed optical networks and Shared Bus Local Area Networks. Retrial queues are characterized by the phenomenon that an arriving customer who finds the server busy upon arrival is obliged to leave the service area and repeat his demand after some time [1, 2, 3, 4, 5].

Service priority is clearly today a main feature of the operation of almost any manufacturing system. This leads us to multi-class retrial queues. Such models are essentially more difficult than the single class models. In contrast, there are a great number of numerical and approximation methods which are of practical importance. One important approach is monotonicity which can be investigated using the stochastic comparison method based on the general theory of stochastic orders. Stochastic comparison methods have been used to produce bounds and approximations for queue length processes, waiting times and busy period distributions in many queueing systems.

The main objective of this work is to use stochastic ordering techniques to establish various monotonicity results for some performance measures of an M/G/1 queue with repeated attempts and two types of calls. We focus on the stochastic comparison theory since it remains a qualitative approach of own interest. After addressing the monotonicity properties of the transition operator of the embedded Markov chain relative to some particular stochastic orders (strong stochastic ordering and increasing convex ordering), we obtain comparability conditions and provide bounds for the stationary distribution and some mean characteristics of the system. Numerical applications, based on simulation, are carried out to support the results.

Keywords: Retrial queue; Two types of calls; Insensitive bounds; Simulation.
2010 Mathematics Subject Classification: 60K25; 60E15.
Mathematical model description

We consider an $M_2/G_2/1$ queueing system which serves two types of calls. In the case of blocking the first type customers can be queued whereas the second type customers must leave the service area but return after some random period of time to try their luck again. The model considered is based on following assumptions:

- Two different types of primary customers arrive according to independent Poisson streams:
  - Priority calls: with rate $\alpha$, service time distribution function $B_1(x)$, first moment $\beta_1^1$ and load $\rho_1 = \alpha \beta_1^1$.
  - Non-priority calls: with rate $\lambda$, service time distribution function $B_2(x)$, first moment $\beta_2^1$ and load $\rho_2 = \lambda \beta_2^1$.
- Priority customers are queued after blocking and then served in some discipline such as FCFS.
- Any non-priority customer who finds the server busy upon arrival leaves the system immediately, to seek service again at subsequent epochs until he finds the server free. The retrial times are assumed to be independent and exponentially distributed with parameter $\mu > 0$.

Flows of primary arrivals, intervals between repeated trials and service times are assumed to be mutually independent.

Main results

Although the performance characteristics of such a model were obtained, they are cumbersome (they include integrals of Laplace transform, solutions of functional equations, etc.) and are not very exploitable from the application point of view (performance evaluation, etc.). It is why we use, in the rest of this paper, the general theory of stochastic ordering (see [1, 2, 3, 4, 5]) to study monotonicity properties of the considered model relative to the strong stochastic ordering and increasing convex ordering. We prove the monotonicity of the transition operator of the embedded Markov chain relative to strong stochastic ordering and convex ordering. We obtain comparability conditions for the distribution of the number of customers in the system. Bounds are derived for the mean performance measures of the considered model. Finally, numerical illustrations are provided to support the results.

References


Uncertainty product for Vilenkin groups

Elena A. Lebedeva

1 Saint-Petersburg Polytechnical University, Russia 2 Saint-Petersburg State University, Russia

E-mail: ealebedeva2004@gmail.com;

Abstract We study a localization of functions defined on Vilenkin groups. Localization is characterized by functional $UC_p$ that is similar to the Heisenberg uncertainty constant for $L^2(R)$-functions. We find an analog of a quantitative uncertainty principle. To justify our definition we use some test functions including scaling and wavelet functions.

Introduction

Initially the notion of uncertainty product was introduced for $f \in L^2(R)$ by W. Heisenber [2] and E. Schrödinger [5]. For periodic functions, it was extended by E. Breitenberger [2]. For some particular cases of locally compact groups (namely a euclidean motion groups, non-compact semisimple Lie groups, Heisenberg groups) the counterpart was suggested in [4]. In [3], we introduced this concept for functions defined on the Cantor group. In this paper, we discuss localization of functions defined on the Vilenkin group.

The Vilenkin group $G = G_p$, $p \in \mathbb{N}$, $p \neq 1$, consists of the sequences

$$x = (x_j) = (\ldots, 0, x_k, x_{k+1}, x_{k+2}, \ldots),$$

where $x_j \in \{0, \ldots, p-1\}$ for $j \in \mathbb{Z}$ and there exists at most a finite number of negative $j$ such that $x_j \neq 0$. The group operation on $G$ is denoted by $\oplus$ and defined as the coordinatewise addition modulo $p$:

$$(z_j) = (x_j) \oplus (y_j) \iff z_j = x_j + y_j \ (\text{mod} \ p) \quad \text{for} \quad j \in \mathbb{Z}.$$ 

We denote by $\ominus$ the inverse operation of $\oplus$. If $x \in G$, then $\ominus x$ denotes the inverse element of $x$. Define a mapping $\lambda: G \rightarrow [0, +\infty)$ by letting

$$\lambda(x) = \sum_{j \in \mathbb{Z}} x_j p^{-j}, \quad x = (x_j) \in G.$$ 

The mapping $x \rightarrow \lambda(x)$ is a one-to-one correspondence taking $G \setminus Q_0$ onto $[0, \infty)$, where $Q_0$ consists of all elements terminating with $p-1$'s. Given $\omega \in G$, the function

$$\chi_\omega(x) = \chi(x, \omega) := \exp \left( \frac{2\pi i}{p} \sum_{j \in \mathbb{Z}} x_j \omega_{1-j} \right)$$

is a group character of $G$. The Fourier transform of a function $f \in L^1(G)$ is defined by

$$Ff(\omega) = \int_G f(x) \overline{\chi(x, \omega)} d\mu(x), \quad \omega \in G.$$ 

The generalized Walsh functions for the group $G$ are defined by $w_n(x) := \chi(\lambda^{-1}(n), x).$

Keywords: Vilenkin group; uncertainty product; uncertainty principle; scaling function; wavelet.
2010 Mathematics Subject Classification: 22B99; 42C40.
Main results

Definition 6. Suppose \( f : G \to \mathbb{C} \), \( f \in L_2(G) \), then a functional

\[
UC_p(f) := V(f)V(Ff), \quad \text{where}
\]

\[
V(f) := \frac{1}{\|f\|_{L_2(G)}^2} \min_{\lambda} \int_G (\lambda(x \oplus \tilde{x}))^2 |f(x)|^2 \, dx,
\]

\[
V(Ff) := \frac{1}{\|Ff\|_{L_2(G)}^2} \min_{\tilde{t}} \int_G (\lambda(t \ominus \tilde{t}))^2 |Ff(t)|^2 \, dt
\]

is called the uncertainty product of \( f \).

Theorem 36. Suppose \( f : G \to \mathbb{C} \), \( f \in L_2(G) \). Then the following inequality holds true

\[
UC_p(f) \geq C, \quad \text{where } C \approx 8.5 \times 10^{-5}.
\]

Theorem 37. Let \( f(x) = \sum_{k=0}^{\infty} a_k w_k(x) \) be a uniformly convergent series. Denote

\[
f_n(x) = \sum_{k=0}^{p^n-1} a_k w_k(x).
\]

Let \( V(f) < +\infty \), \( V(Ff) < +\infty \). Then \( UC_p(f) = \lim_{n \to \infty} V(f_n)V(Ff_n) \), where

\[
V(f_n) = \frac{\min_{k_0=0,p^n-1} \sum_{k=0}^{p^n-1} |b_{\lambda^{-1}(k) \oplus \lambda^{-1}(k_0)}|^2 ((k+1)^3 - k^3)/3}{\sum_{k=0}^{p^n-1} |a_k|^2},
\]

\[
V(Ff_n) = \frac{\min_{k_1=0,p^n-1} \sum_{k=0}^{p^n-1} |a_{\lambda^{-1}(k) \oplus \lambda^{-1}(k_1)}|^2 ((k+1)^3 - k^3)/3}{\sum_{k=0}^{p^n-1} |a_k|^2},
\]

and \( b_k = \sum_{s=0}^{p^n-1} a_s w_k(\lambda^{-1}(s/p^n)) \), \( 0 \leq k \leq p^n - 1 \), is the inverse discrete Vilenkin-Chrestenson transform.

Acknowledgments

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References


Hybrid finite element formulation of mild slop equation in a rectangular domain

Prashant Kumar¹, Rupali Gupta¹

¹National Institute of Technology Delhi, India
E-mail: ¹prashantkumar@nitdelhi.ac.in; ¹rupali@nitdelhi.ac.in

Abstract   In the present study, mild slope wave equation is used to resolve the problem wave induced oscillation in a harbor. The mild-slope equation is an effective approximation for treating the combined effects of refraction and diffraction of water waves. The solution of mild slope equation is determined by using Hybrid Finite Element (HFEM) method for complex geometrical domain including the partial reflection boundary condition. The amplification factor is determined to analyze the resonance modes in a rectangular harbor. Convergence analysis is conducted for the validation of numerical scheme. Further, proposed numerical approach is compared with previous existing study conducted by Lee (1971). The proposed numerical scheme can be implemented on the complex geometrical domains such as ports and harbor.

References


Keywords: Hybrid finite element; mild slope equation; diffraction and partial reflection; resonance modes; convergence analysis.

2010 Mathematics Subject Classification : 65N30; 65L60; 76B15; 35Q35.
Numerical Simulation of a High-Current Z-Discharge in water, initiated by an electrical Explosion of a thin-walled metal Shell

V. Antonov 1, N. Kalinin 2, A. Kovalenko 2

1 Peter the Great St.Petersburg Polytechnic University, Russia
2 Joffe Physical-Technical Institution, Russian Academy of Science, Russia

E-mail: 1antonovvi@mail.ru

Abstract Powerful electric discharges in plasma channels generated in liquid are widely used in scientific research, technology and development of modern technologies [1]. Much of the applied works, which explores the powerful electric discharges in liquid, are aimed at harnessing the power of hydrodynamic impulses for materials processing. The ability to achieve high efficiency of energy conversion provides a wide range of technological applications of electrical discharges. Among them, crushing, pressing and molding materials by shock waves. In order to study redistribution of energy expansion in a channel developed a series of simplified discharge models that are useful for solving a particular problem [1][2].

Introduction

Our scientific interest to the powerful electrical discharge in the water caused by the idea of getting supercritical water conditions under which it acquires properties effective solvent in a broad enough area, adjacent to the category. The need to study the dynamics of this area in development level and determination of optimal operating conditions, enabling you to get physical-chemical condition of supercritical fluid water required for the implementation of a number of technological processes.

Main results

A mathematical model of a high-current low-temperature Z-discharge in a liquid, forming by the electrical explosion of a thin-walled metal shell, connected to a pulsed high-voltage generator through a long line, has been developed. The bases of the model consists in the one-dimensional magneto hydrodynamic approximation, in which the change of the physical state of a substance is described in coordination with the physical processes occurring in the system and in the surrounding fluid. Allowances are made for the basic processes inherent in the electrical discharges - the diffusion of the magnetic field and the high-power pulsed heating of the material, its ionization, contraction and expansion, the generation of shock waves, the energy dissipation due to thermal conductivity and viscosity, radiation transport in the radiant heat transfer approximation. For the correct description of the evolution of the conductor physical state and the surrounding liquid, a wide-range semi empirical models for solving the equations of state and transport coefficients are used. That allows considering the variation of parameters from the normal values to values significantly above the critical points of metals and liquids. The completely conservative difference scheme of through calculation is used for the numerical solution of one-dimensional system of magneto hydrodynamics equations, presented in Lagrangian form. The above model was tested by results of experimental data.

Keywords: mathematical model, Z-discharge, electrical explosion, metal shell, effective solvent.

2010 Mathematics Subject Classification: 65C20; 74A15; 82D10.
The temperature of plasma discharge in the channel can reach tens of thousands of degrees and the radiant heat transfer in the balance of power begins to play a significant role. To describe the role of radiation in the continuous spectrum (brake and recombination) in the balance of power is used a simplified model that is based on the assumption that, if the length of the distance photons, average for Planck, is the more than the characteristic dimension of the radiating area, the radiation has a 3-d character. If the average mileage of the photons is less than the characteristic dimension the task, heat transfer by radiation can be expected in the approximation of radiant heat. Required for these calculations the charged plasma composition is calculated by the Saha model taking into account the lower capacity ionization. The average mileage of the photons for Planck and Rosseland determined in accordance with the model. In typical conditions of powerful electric discharges it is known that up to temperature $T$ less than 20000 K the water is optically transparent and plasma cooling by radiant emission can be expected in the approximation of three-dimensional radiation, and at $T$ more than 20000 K - in approximation of radiant heat.

References


A fourth order B-spline collocation method for numerical simulation of periodic Burgers' equation

Ramesh Chand Mittal, Rajni Rohila

Department of Mathematics, Indian Institute of Technology, Roorkee- 247667, Uttarakhand, India

E-mail: rcmmfma@iitr.ac.in; driit.dma2014@iitr.ac.in

Abstract  A fourth order numerical method based on cubic B-spline functions has been proposed to solve periodic Burgers’ equation. Four well known problems of periodic Burgers’ equation have been considered to test the efficiency of the method. Approximate solutions are in good agreement with the earlier studies. We have aimed to develop a computationally efficient numerical method for solutions of pde's. With small modifications, method can be employed to solve different kind of partial differential equations arising in various areas of science and engineering.

References


Keywords: Periodic Burgers' equation; Crank–Nicolson method; B-spline functions; collocation method.
2010 Mathematics Subject Classification: 45M15; 65D07.
Pseudo almost periodic and pseudo almost automorphic solutions of neutral delay integral equation

Farouk Chérif¹, Chaouki Aouiti²

University of Sousse, Tunisia ²University of Carthage, Tunisia

E-mail: ¹faroukcheriff@yahoo.fr; ²chaouki.aouiti@fsb.rnu.tn

Abstract  This paper deals with the existence and uniqueness of pseudo-almost periodic and pseudo almost automorphic solutions to some neutral delay integral equations of advanced type.

References


Keywords: Integral equation; delay; pseudo almost periodic; pseudo almost automorphic; fixed points.

2010 Mathematics Subject Classification: 35B15; 47H10.
Numerical Approaches for the Epistemic Uncertainty Analysis of Queues

Sofiane Ouazine¹, Karim Abbas ²

¹Bejaia University, Algeria  ²Bejaia University, Algeria

E-mail: ¹wazinesofi@gmail.com; ²kabbas.dz@gmail.com

Abstract  When we model a real system by a queueing model, we suggest that all parameters of the underlying model are seldom perfectly known. In this paper we investigate the numerical evaluation of stationary characteristics of the M/M/1/N queue where we suppose that the service rate and the inter-arrival rate are not assessed in perfect manner, i.e. they are subject to propagate uncertainty. Therefore, we introduce a new queueing model where we take into consideration the factor of the error in the prediction of such parameters. This paper proposes a numerical approach based on Taylor series expansion for analyzing the stationary characteristics of the considered queueing models. Additionally, approximate expressions of the probability density functions, the expectation and the variance of the stationary characteristics are obtained and compared to the corresponding simulations results.

Introduction

Markovian models are well-known as a power modeling tool for analyzing a variety of real-problems that we can describe by means of Markov chain. In a such case, the stationary distribution of the underlying Markov chain has a key role in the performance evaluation of such models, because we can deduce the majority of others stationary characteristics in terms of the stationary distribution. Commonly, when we model a real system by a queueing model, we suggest that all parameters of the underlying model are seldom perfectly known. Suppose, for example, we want to model the dynamic of a single server queue with a finite capacity and service discipline First-Come First-Served (FCFS). Even if this queueing model is considered as a more realistic model which is defined as the closest model to the real one, this paper introduces a new queueing model where we take into consideration the factor of the error in the prediction of the service rate and the inter-arrival rate. However, an analysis of a sample of service times or a sample of the inter-arrival times indicates that they are not quite independent or not quite exponentially distributed. These deviations observed in the inter-arrival times and in the service times may be due to deficient statistical data or sampling error. Moreover, in this case, the type of distribution is known, its parameters are still assessed with a statical uncertainty.

In this paper, we investigate the numerical evaluation of stationary characteristics of the M/M/1/N queue, with an emphasis on perturbation analysis where we suppose that the service rate \( \mu \) and the inter-arrival rate \( \lambda \) are not assessed in perfect manner, i.e. they are subject to propagate uncertainty. More precisely, both parameters become random variables, \( \lambda = \lambda(\varepsilon_1) \) and \( \mu = \mu(\varepsilon_2) \), where \( \varepsilon_1 \) and \( \varepsilon_2 \) are two normal random variables representing the white noises on computing the two parameters \( \lambda \) and \( \mu \) respectively. Therefore, we consider the stationary distribution of the queue-length in the M/M/1/N queue as a transformation of random variables, in notation \( \pi(\varepsilon_1, \varepsilon_2) \). This paper proposes a numerical approaches based on Taylor series expansion for analyzing the stationary characteristics of the new queueing model. Additionally, we approximate expressions of the probability density functions, the expectation and the variance of the stationary characteristics. The obtained numerical results are compared to the corresponding simulations results.

Keywords: Queues; Parametric uncertainty; Taylor series expansions; Simulation.

2010 Mathematics Subject Classification: 68T37; 60K25.
The rest of the paper is organized as follows. In Section 2, we present closed-form expressions for the high-
sensitivity of the Markov chain stationary distribution with respect to model parameters in terms of the funda-
mental matrix (see e.g. [1][2][3]). These formulas are used to compute the coefficients of the Taylor series expansions
of the stationary distribution. In Section 3, we introduce an approach, namely Taylor Series Expansions (TSE) for
computing the probability density functions of the stationary distribution. The new queueing model (M/M/1/N
queue with parametric uncertainty) is described in Section 4. In Section 5, we provide a numerical results obtained
by the implementation of the proposed approach to approximate expressions of the probability density functions
(pdfs), the expectation and the variance of the stationary characteristics. These results compared to the corre-
sponding simulations results. Finally, some concluding remarks are given in Section 6.

Main results

The main results of this contribution is the following theorem:

**Definition 7.** We assume that \( f \in \mathcal{C}^n(\Omega) \), where \( \Omega \) is an open set of \( \mathbb{R}^m \). In order to express higher-order derivatives more efficiently, we introduce the following multi-index.

A multi-index is a vector \( h = (h_1, \ldots, h_m) \) where each \( h_i \) is a nonnegative integer. The order of the multi-index is \( |h| = h_1 + \ldots + h_m = n \). Given a multi-index \( h \), we define:

\[
D^h f = \frac{\partial^n f}{\partial \theta_1^{h_1} \partial \theta_2^{h_2} \ldots \partial \theta_m^{h_m}} = \frac{\partial^{h_1}}{\partial \theta_1^{h_1}} \frac{\partial^{h_2}}{\partial \theta_2^{h_2}} \ldots \frac{\partial^{h_m}}{\partial \theta_m^{h_m}} f.
\]

In the following theorem, we state the higher-order sensitivity of the stationary distribution in terms of the funda-
mental matrix with respect to the vector \( \theta = (\theta_1, \theta_2, \ldots, \theta_m) \).

**Theorem 38.** Provided that all components of \( P_{\theta} \) are a functions of \( \mathcal{C}^n(\Omega) \), where \( \Omega \) is an open set of \( \mathbb{R}^m \). Then, the partial derivatives of the stationary distribution \( \pi_{\theta} \) ( \( \theta \) is an interior point of \( \Omega \)) of an ergodic finite-state Markov chain with respect to \( \theta \) are given by:

\[
D^h \pi_{\theta} = \sum_{|i_1, i_2, \ldots, i_m| < n} C_{i_1}^{h_1} C_{i_2}^{h_2} \ldots C_{i_m}^{h_m} \left( \frac{\partial^{(i_1 + i_2 + \ldots + i_m)}}{\partial \theta^{(i_1, i_2, \ldots, i_m)}} \pi_{\theta} \right) \left( \frac{\partial^{(h_1 - i_1 + h_2 - i_2 + \ldots + h_m - i_m)}}{\partial \theta^{(h_1 - i_1, h_2 - i_2, \ldots, h_m - i_m)}} P_{\theta} \right) Z_{\theta},
\]

where \( ||h||_1 = n \).

References


Novelty Fractional–order Difference Application in Systems Modelling

Dorota Mozyrska¹, Piotr Ostalczyk²

¹ Bialystok University of Technology, Poland ² Lodz University of Technology, Poland

E-mail: ¹d.mozyrska@pb.edu.pl; ²postalcz@p.lodz.pl

Abstract  In the paper a linear time–invariant difference equation is applied to model a dynamic system. The forward difference with higher fractional–order (constant and variable) is used. The fundamental structure of the equation is proposed. The investigations are illustrated by numerical examples and a real simple electrical circuit modeling.

Introduction

The fractional calculus for many years is an important and recognised tool in systems modelling. Usually, the fractional order derivatives are approximated by the fractional–order differences, see [1, 2, 3]. Having measured signal sequence we can use the forward fractional difference to improve the signal dynamic properties. There are many problems to contend with new model. Our main goal is to show how the model approximates measured signal. We try too look into proper models and compare their coefficients. From our investigations we conclude that models with higher fractional orders are more malleable in modelling of some type of systems.

Main results

In the paper we take into account fractional orders that are greater than one, as well those with variable orders. For information stability and order reduction for various fractional differences see [4]. In the case of linear fractional constant order difference systems the $\mathcal{Z}$-transform can be used as an effective method of analysis of systems. However in the case of variable order it is no longer possible. We show that in case of constant fractional–order one can reduce the order of the considered systems by transforming them to the systems with the partial orders from the interval $[0, 1]$. Then the results can be applied to transformed systems and consequently, we can get the conditions for stability of linear difference systems with fractional orders $\alpha+1$, where $\alpha \in (0, 1]$. We propose also an equivalent but very useful vector–matrix description of the variable–, fractional–order integrator is applied. It becomes a great tool in the variable–, fractional–order integrators description. We examine the thesis that and show in examples that modelling by equations with fractional order gives some improvement with respect to the integer order models.

Acknowledgments

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Keywords: discrete linear system; fractional calculus; system modeling.

2010 Mathematics Subject Classification: 26A33; 34A60.
References


Integration of Heterogeneous Biological Data for Protein Function Prediction

Natalia Novoselova¹, Igor Tom²

¹,² United Institute of Informatics Problems, Belarus

E-mail: ¹novosel@newman.bas-net.by; ²tom@newman.bas-net.by

Abstract  We describe the approach for integrating different sources of biological information for protein function prediction. The approach is based on the unified representation of the data by means of kernel functions, including protein-protein interaction (PPI) data, protein domain (PFAM) information and protein localization data. We have applied the support vector machine (SVM) classifier with individual kernels and integrated kernel similarity matrices to estimate the functional prediction performance of yeast genes, using Gene Ontology (GO) terms. The experiments show that integrating data sources improves the prediction, especially by weighting the data types according to its importance. We analyze the dependence of prediction performance of the individual functional terms on the kind of integrated data and propose the procedure for choosing the best combination of data sources.

References


Keywords: support vector machine; diffusion kernel; protein function prediction; cross-validation.

2010 Mathematics Subject Classification: 00A69; 62P10; 62H30; 68T10; 92C42; 68-04.
Approximate Lie Point Symmetries for Fractional Partial Differential Equations

Stanislav Yu. Lukashchuk

Ufa State Aviation Technical University, Russia

E-mail: lsu@ugatu.su

Abstract  Partial fractional differential equations in which the order of fractional differentiation is close to an integer number is considered. A small parameter is introduced in such equations, and expansions of fractional derivatives into a power series in this small parameter is obtained. A possibility of approximation of a fractional differential equation by a differential equation of integer order with a small parameter is discussed. It is shown that the theory of approximate transformation groups can be used for approximate symmetries calculating of such approximate equations. Using approximate symmetries, corresponding approximate invariant solutions can be constructed. These solutions can be also considered as approximate solutions of initial fractional differential equations. The proposed approach is illustrated by an example of subdiffusion equation with the Riemann–Liouville time-fractional derivative.

Introduction

Partial fractional differential equations are widely used as mathematical models of anomalous processes. In applications, an assumption that the order of fractional differentiation is close to an integer number is frequently valid. Then a small parameter can be introduced into equation. For example, such approach was used in [1] for constructing approximate solutions to several fractional differential equations.

It is known (see, e.g., [2]) that the fractional integral \(a_\varepsilon f(x)\) with a small parameter \(\varepsilon \) \((0 < \varepsilon \ll 1)\) can be represented as

\[
a_\varepsilon f(x) = f(x) + \varepsilon \left(-\Gamma'(1) f(x) + \frac{d}{dx} \int_a^x \ln(x-t)f(t)dt\right) + o(\varepsilon).
\]

Using (139), the corresponding expansions for fractional derivatives can be obtained. Then a fractional differential equation is approximated by an equation with a small parameter, and various perturbation methods can be used for solving this equation.

Approximate Lie group analysis (see, e.g., [3]) is a powerful perturbation technique for investigation qualitative properties of differential equations with a small parameter. Nevertheless, its application to approximations of fractional differential equations that obtained by using (139) is very nontrivial due to nonlocality of such equations. A proposed approach is based on an alternative expansion of fractional derivatives into a power series in a small parameter.

Keywords: fractional derivative; fractional partial differential equation; small parameter; approximate symmetry; approximate invariant solution.

2010 Mathematics Subject Classification: 35R11; 35B20; 70G65.
Main results

Let $\varepsilon$ be a small parameter. The following theorem is proved.

**Theorem 39.** If function $f(x)$ is analytic in $(a, b)$ ($-\infty < a < b < \infty$) then at every point $x \in (a, b)$ such that $\varepsilon|\ln(x - a)| = O(\varepsilon)$ the following expansion is valid:

$$aD_x^{\alpha, \varepsilon} f = f^{(n)}(x) \pm \varepsilon \left\{ \psi(n + 1) - \ln(x - a) \right\} f^{(n)}(x) - \sum_{k=0}^{\infty} \left( \frac{(-1)^{k-n}}{(k-n)!} n! (x - a)^{k-n} f^{(k)}(x) \right) + o(\varepsilon), \quad (140)$$

where $aD_x^{\alpha, \varepsilon} f$ is the left-sided Riemann–Liouville fractional derivative \[2\], $n > 0$ is an integer number, and $\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$ is the digamma function.

Similar theorems are proved for the right-sided Riemann–Liouville fractional derivative and the Caputo fractional derivatives.

**Example 1.** Let us consider a nonlinear time-fractional subdiffusion equation

$$\frac{\partial}{\partial t} u = \left( e^{u} u x \right)_x, \quad u = u(t, x), \quad \alpha \in (0, 1). \quad (141)$$

If $\alpha = 1 - \varepsilon$ ($0 < \varepsilon \ll 1$) then, in view of (140), this equation can be approximated by the following equation with a small parameter:

$$u_t + \varepsilon \left( \ln t + \gamma - 1 \right) u_t + \frac{u}{t} + \sum_{n=1}^{\infty} \frac{(-1)^{n} t^n}{n(n+1)!} D_t^{n+1} u \right) = (e^{u} u_x)_x. \quad (142)$$

Approximate Lie point symmetries for this equation can be found using the classical algorithm (see, e.g., \[3\]). For example, the equation (142) has the following stable approximate symmetry:

$$X = [1 - \varepsilon(\ln t - 2)] \frac{\partial}{\partial t} - [1 - \varepsilon \ln t] \frac{\partial}{\partial u} \quad (143)$$

Note that this symmetry does not have an analogue in the set of exact point symmetries of fractional subdiffusion equation (141).

The approximate invariant solution of the equation (142) corresponding to (143) has the form

$$u(t, x) = \left( 2\varepsilon - 1 \right) \ln t + \ln \left( C_2 + C_1 x - \frac{x^2}{2} \right) + \frac{2 - \gamma}{2} \int \int \ln \left( C_2 + C_1 x - \frac{x^2}{2} \right) dxdx + \frac{\int \int \ln \left( C_2 + C_1 x - \frac{x^2}{2} \right) dxdx}{C_2 + C_1 x - \frac{x^2}{2}},$$

where $C_1 = C_1(\varepsilon)$, $C_2 = C_2(\varepsilon)$ are arbitrary constants for a given $\varepsilon$.

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References

The simulation of dispersion of particles in a planar mixing layers in depend on inlet effects

Altyn Makasheva\textsuperscript{1}, Altynshash Naimanova\textsuperscript{3}

\textsuperscript{1,2} Institute of Mathematics and Mathematical Modeling, Kazakhstan

E-mail: \textsuperscript{1}altyn-mak@mail.com; \textsuperscript{2}naimanova@yahoo.com

Abstract Numerical simulations of a particle dispersion in a planar mixing layer dominated by large scale vertical structures are reported based on the Euler-Lagrangian approach. The two-dimensional time-dependent multi-species gas-phase Navier-Stokes equations are solved numerically using the high-order essentially non-oscillatory (ENO) scheme for the high-speed mixing layer. The dispersion of particles is explored by following their trajectories in the mixing layer. A detailed numerical study of dispersion characteristic of particles in depend on large-scale coherent structures of multispecies gas is performed. Also, the influence of sizes and injection location relative to the splitter plate on the particle dispersion is studied.

Introduction

The mixing of two layers with different velocity and particle loading profiles is commonly observed in natural and industrial processes. Shear layer with particles formed by the confluence of two flows are physically complex, which requires the simultaneous resolution of particle-turbulence and gas-particle interactions. Such flows are explored experimentally and numerically \cite{1,2,3,4}. As a rule, the experiments are very complex and expensive. Therefore, at the moment, the numerical simulation of particle dispersion in a mixing layer is the most preferable.

Numerically, there are two main modeling approaches which are suitable for the simulation of these flows \cite{3,4}. The first approach models both the carrier phase and the particle phase in the Eulerian frame \cite{3}. The particle phase is governed by conservation laws similarly to the carrier phase.

Another approach for modeling two-phase flows is a mixed Eulerian-Lagrangian viewpoint \cite{4}. In these models, particle paths are traced in the Lagrangian reference frame while the fluid governing equations are solved in the Eulerian frame.

The purpose of this paper is the study of particle dispersion in a plane developing mixing layer in Eulerian-Lagrangian formulation. The investigation focuses on the effects of developing vortex structures on particle dispersion of different size in a mixing layer.

Numerical model and main results

Within the scope of this study we use a two-dimensional direct numerical simulation (2D-DNS) for vortex structures of multispecies gas flow. The unsteady, planar compressible Navier-Stokes equations are considered for multi-species gas mixture.

Keywords: two-phase flows; turbulent gas-particle flows; ENO scheme; Eulerian-Lagrangian approach; mixing layer; supersonic flow; particle dispersion.

2010 Mathematics Subject Classification: 76T99; 76J20; 76F65.
The particles are traced assuming one-way coupling between the continuous and the dispersed phases, i.e. the particles are influenced by the gas phase. Consequently, governing equations for particles are equations of trajectory, momentum and temperature.

To close the systems of the original equations, we use the Wilke formula to determine the mixture viscosity coefficient in terms of the mass fractions [6].

For the gas at the inflow, all physical variables are varied smoothly from hydrogen (fuel) flow to air flow using a hyperbolic-tangent function [7]. On the lower boundary, upper boundary and at the outflow the non-reflecting boundary conditions are imposed. In order to produce the roll-up and pairing of vortex rings, an unsteady boundary condition is also applied at the inlet plane [7]. The calculation of the Navier-Stokes equations is performed with the use of the Nav2D code [8] based on the third-order ENO scheme.

The particles are injected randomly with the velocity value equal to the gas velocity at the inflow boundary (x=0, t=0) and from different injection location relative to the splitter plate.

The influence of large-scale coherent structures in a spatially-developing mixing layer on the particle dynamics is numerically studied. Detailed analysis reveals that the intermediate size particles are caught by them, leading to their enhanced dispersion, while the large particles are mostly unaffected by the large eddies. However, the asymmetric entrainment of particles on the periphery of the mixing layer is presented for intermediate and large sizes. Also, the numerical experiments reveal that the involving of particles in the vorticity structures strongly depends on transver injection location.

References


Linear Stability of a Chemically Reacting Fluid in Tall Vertical Containers

Andrei Kolyshkin

Riga Technical University, Latvia

E-mail: andrejs.koliskins@rtu.lv

Abstract  Linear stability of a convective flow of a viscous incompressible fluid in a tall vertical cylindrical container is investigated. Steady convective motion is generated by internal heat sources distributed in the fluid in accordance with the Arrhenius law. Nonlinear boundary value problem describing the base flow is solved numerically. Linear stability is investigated with respect to axisymmetric and asymmetric disturbances in a circular pipe and annulus. Numerical results show that for some values of the parameters marginal stability curves consist of two separate branches. It is shown that the flow is destabilized as the Prandtl number and the Frank-Kamenetskii parameter increase.

Introduction

Stability analysis of a chemically reacting fluid is investigated in [1]-[3]. There are many factors which affect the structure of the flow so that some simplifying assumptions are usually made in order to analyze the problem. The analysis of thermal effect of the reaction is important in applications [4]. Consider a tall vertical cylindrical channel (a circular pipe or an annulus) filled with a viscous incompressible fluid. The channel is closed so that the total fluid flux through the cross section of the channel is zero. Internal heat sources are distributed within the fluid in accordance with the Arrhenius law [5] as a result of exothermic reaction:

\[ \dot{Q} = Q_0 k_0 \exp \left[ -\frac{E}{RT} \right], \]  

(144)

where \( \dot{Q} \) is the absolute temperature, \( R \) is the universal gas constant, and \( Q_0, k_0 \) and \( E \) are the parameters of the chemical reaction. Linear stability of the convective motion due to internal heat generation in accordance with (144) is investigated in this paper.

Main results

The system of equations describing thermal convection under the Boussinesq approximation has the form

\[ \frac{\partial v}{\partial t} + Gr (\nabla v) v = \nabla p + \Delta v + T k, \]  

(145)

\[ \frac{\partial T}{\partial t} + Gr v \nabla T = \frac{1}{Pr} \Delta T + \frac{F}{Pr} \exp \left[ \frac{T}{1 + bT} \right], \]  

(146)

\[ \nabla v = 0, \]  

(147)

Keywords: linear stability; collocation method; Arrhenius law; chemically reacting fluid.

2010 Mathematics Subject Classification: 76E05; 80A32; 80M22; 65L10.
where \( \mathbf{v} = (v_r, v_\phi, v_z) \) is the velocity vector written in the cylindrical coordinate system \((r, \phi, z)\), \( T \) is the dimensionless temperature deviation from the wall temperature, \( p \) is the pressure, \( b = RT_0/E \). Consider a steady convective motion of the following structure

\[
\mathbf{v} = (0, 0, V_0(r)), \quad T = T_0(r), \quad p = Az + B, \tag{148}
\]

where \( r \) and \( z \) are the radial and vertical coordinates, respectively, and \( A \) and \( B \) are constants. The nonlinear boundary value problem for the determination of the base flow \( V_0(r) \) and \( T_0(r) \) is solved numerically using Matlab. Linear stability of the base flow is investigated with respect to axisymmetric and asymmetric perturbations by means of a collocation method based on the Chebyshev polynomials. Three parameters characterize the problem - the Grasshof number, the Prandtl number and the Frank-Kamenetskii parameter. Numerical results show that marginal stability curves consist of two separate branches with two local minima. It is shown that the increase of the Prandtl number and the Frank-Kamenetskii parameter destabilizes the flow.

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**References**


Integrals of systems of two second-order ODEs admitting four-dimensional Lie algebras

A.A. Gainetdinova¹, R.K. Gazizov²
¹,² Ufa State Aviation Technical University, Russian Federation

E-mail: ¹aliya-oct@yandex.ru; ²gazizovrk@gmail.com

Abstract We suggest the algorithm for constructing the first integrals of systems of two second-order ordinary differential equations admitting four-dimensional Lie algebras of operators. We claim that if admitted transformation group has two second-order differential invariants, the first integral of corresponding system can be obtained by using invariant representation and operator of invariant differentiation.

Introduction

We consider systems of the form

\[
\begin{aligned}
    x'' &= f(t, x, y, x', y') , \\
    y'' &= g(t, x, y, x', y') 
\end{aligned}
\]  

(149)

admitting four-dimensional (real) Lie algebras with generators

\[ X_i = \xi_i(t, x, y) \frac{\partial}{\partial t} + \eta_i(t, x, y) \frac{\partial}{\partial x} + \zeta_i(t, x, y) \frac{\partial}{\partial y}, \quad i = 1, 2, 3, 4, \]  

(150)

and suggest method to construct the first integrals of them. This method is based on using invariant representation of (149) and operators of invariant differentiation.

For using the invariant representation of (149), we construct the differential invariants of admitted transformation groups up to second order. The \(k\)-order invariants are obtained from the system of linear equations

\[ X_i, k(I_k) = 0, \]  

(151)

where \(k\) is the order of prolongation, \(X_{i, k}\) is the prolongation of the operator \(X_i\) up to \(k\)-th derivatives.

Also we use the invariant differentiation operators. Action of such operator to \(k\)-order invariants gives \((k+1)\)-order invariants (see, e.g., [1]). In the case of one independent variable and two dependent variables, the operator has the form \(\lambda(t, x, y, x', y')D_t\), where \(D_t\) is the total differentiation operator.

In [1] it was shown that the operator of invariant differentiation \(\lambda D_t\) can be obtained from the condition of commutation

\[ [\lambda D_t, X_i, \infty] = 0, \quad i = 1, \ldots, r. \]

It is easy to show (see, e.g. [2]) that \(\lambda\) satisfies to the following system of equations:

\[ X_i, \infty(\lambda) = \lambda D_t(\xi_i), \quad i = 1, \ldots, r, \]

where \(r\) is dimension of Lie algebra.

Keywords: systems of two second-order ODEs, four-dimensional Lie algebras of operators, differential invariants, operator of invariant differentiation.

2010 Mathematics Subject Classification: 34A99; 76M60; 70H33.
Main results

The invariant representation of system (149) is

\[ I^{(1)}_2 = F(I), \quad I^{(2)}_2 = G(I), \]  

(152)

where \( F \) and \( G \) are arbitrary functions, \( I^{(i)}_2, i = 1, 2 \) are second-order differential invariants, \( I \) is a first-order differential invariant or algebraic invariant.

By the definition of the operator of invariant differentiation, we obtain

\[ \lambda D_t(I) = \Theta(I, I^{(1)}_2, I^{(2)}_2) \]

with some function \( \Theta \). On the solutions of (152), it satisfies the equation

\[ \lambda D_t(I) \big|_{152} = \Phi(I), \]  

(153)

where \( \Phi(I) = \Theta(I, F(I), G(I)) \).

Equation (153) can be rewritten in the form

\[ \frac{dI}{\Phi(I)} = \frac{dt}{\lambda(t, x, y, x', y', \ldots)}. \]  

(154)

Obviously, the left hand side of this equation can be integrated by quadratures. It is easy to show that for all canonical forms of Lie algebras \( L_4 \) of generators (150) (see, e.g. [3]) and for corresponding canonical systems, the right hand side of (154) is integrated by quadratures. Arbitrary change of variables keeps the integrability of the right hand side of (154) (see, e.g. [4]), i.e. the following theorem is valid.

**Theorem 40.** Let system (149) admit a four-dimensional Lie algebra with generators (150). If the system can be represented by differential invariants, then it has a first integral. This integral is obtained from (154). Moreover, system (149) is integrated by quadratures.

This algorithm can be generalized for integration of the systems of \( k \) \( m \)-order ODEs, \( km \geq 4 \), as well as such systems with a small parameter, which is invariant with respect to approximate symmetries, and systems of fractional differential equations.

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References


Diversity Normed Spaces

Shohreh Golpaiganifard¹, Kourosh Nourouzi²

¹² K. N. Toosi University of Technology, Iran

E-mail: ¹shohregolpaigani@gmail.com; ²nourouzi@kntu.ac.ir

Abstract  Motivated by the concept of diversities given in [1], we introduce diversity normed spaces which are a generalization of normed spaces. In particular, we show that every diversity normed space can be densely embedded in a diversity Banach space. Furthermore, we show that every injective diversity normed space is hyperconvex.

Introduction

Let $X$ be a nonempty set and $\langle X \rangle$ denote the family of all finite subsets of $X$. A diversity is a pair $(X, \delta)$, where $\delta : \langle X \rangle \to \mathbb{R}$ is a function satisfying the following properties for all $A, B, C \in \langle X \rangle$:

(D1) $\delta(A) \geq 0$;

(D2) $\delta(A) = 0 \iff |A| \leq 1$;

(D3) If $B \neq \emptyset$, then $\delta(A \cup C) \leq \delta(A \cup B) + \delta(C \cup B)$ (see [1]).

In this talk, motivated by the concept of diversities we introduce diversity normed spaces. Among various examples and results, we also give the following two main results.

Main results

Theorem 41. Every subadditive diversity normed space can be densely embedded in a diversity Banach normed space.

Theorem 42. Every injective diversity normed space is hyperconvex.

References


Keywords: diversity; diversity norm.

2010 Mathematics Subject Classification: 46B40; 47L25.
On Error Estimates for Approximate Solutions of Certain Discrete Equations

Alexander Vasilyev
Belgorod National Research University, Russia

E-mail: alexvassel@gmail.com

Abstract We consider discrete equations generated by Calderon–Zygmund singular integral operators. For such equations we obtain an approximate solution and give a comparison between continue and discrete cases. An error estimate for approximate solution is given in appropriate discrete functional spaces and solvability conditions for discrete equations are described.

Introduction

We consider the following integral equation

$$a(x)u(x) + v.p.\int_D K(x, x - y)u(y)\,dy = v(x), \quad x \in D,$$ \hspace{1cm} (155)

where $K(x, y)$ is a Calderon–Zygmund kernel with some smoothness properties, $D$ is a certain canonical domain $\mathbb{R}^m$ or $\mathbb{R}^m_+ = \{x \in \mathbb{R}^m : x = (x', x_m), x_m > 0\}$, $a(x), v(x)$ are given functions.

We study a discrete analogue of the equation (155)

$$a_d(\tilde{x})u_d(\tilde{x}) + \sum_{\tilde{y} \in h\mathbb{Z}^m \cap D} K_d(\tilde{x}, \tilde{x} - \tilde{y})u_d(\tilde{y})h^m = v_d(\tilde{x}), \quad \tilde{x} \in h\mathbb{Z}^m \cap D,$$ \hspace{1cm} (156)

for a function $u_d$ of a discrete variable $\tilde{x}$ where the index “d” denotes a restriction of given functions $a, K, v$ on lattice points, $h > 0$.

For these equations one defines symbols

$$\sigma(x, \xi) = a(x) + \lim_{\varepsilon \to 0, N \to \infty} \int_{|x| < N} K(x, y)e^{-i\varepsilon \cdot \xi}\,dy,$$

$$\sigma_d(\tilde{x}, \tilde{\xi}) = a_d(\tilde{x}) + \sum_{\tilde{y} \in h\mathbb{Z}^m \cap D, \tilde{y} \neq 0} K_d(\tilde{x}, \tilde{y})e^{i\tilde{y} \cdot \tilde{\xi}}h^m,$$

and using its properties one studies a solvability of the discrete equation (156) and a comparison between solutions of equations (155) and (156) for that cases when $a(x)$ and $K(x, y)$ do not depend on $x$.

Keywords: discrete equation; approximate solution; error estimate.
2010 Mathematics Subject Classification: 45E10; 65R20.
Main results

Our main results are related to particular case of equations (155) and (156) namely when symbols of equations (155) and (156) do not depend on \(x(\sigma(x,\xi) \equiv \sigma(\xi), \sigma_d(\tilde{x},\xi) \equiv \sigma_d(\xi))\), and we formulate the following result for this case only.

**Theorem 43.** Let \(\sigma(\xi)\) be a differentiable function on \(\mathbb{R}^m \setminus \{0\}\). Then following assertions hold.

- If \(\sigma(0,\cdots,0,-1) = \sigma(0,\cdots,0,+1)\) then equations (155) and (156) are uniquely solvable or unsolvable simultaneously in spaces \(L_2(D)\) and \(L_2(h\mathbb{Z}^m \cap D)\) respectively, \(\forall h > 0\).

- If the right-hand side \(v\) of the equation (155) satisfies a Hölder condition of order \(0 < \alpha < 1\), and \(v_d\) is a restriction of \(v\) on \(h\mathbb{Z}^m \cap D\), then for solutions of (155) and (156) one has the following estimate

\[
|u(\tilde{x}) - u_d(\tilde{x})| \leq ch^{\alpha} \ln \frac{1}{h}
\]

with a constant \(c\) non-depending on \(h\).

Some estimates for finite approximations of the equation (156) are obtained also [4].

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References


The Moving Front Solution in a Two Dimensional Problem from Reaction-Diffusion-Advection Equation

Evgeny Antipov , Natalia Levashova , Nicolay Nefedov

Faculty of Physics, M.V. Lomonosov Moscow State University, Moscow, Russia

E-mail: a.evgen.a@gmail.com

Abstract The conditions for the existence of a solution in a form of a two-dimensional moving front for the reaction-advection-diffusion problem in the case of small diffusion coefficient and large advective term are formulated. An asymptotic approximation in powers of a small parameter of such a solution is constructed. The proof of the existence of a solution approximated by the asymptotics constructed is provided. For the last purpose the asymptotical method of differential inequalities is used

Introduction

\[ \epsilon \Delta u - \frac{\partial u}{\partial t} = \{A(u, x, y), V\} u + B(u, x, y), \quad y \in (0, a), \quad x \in (-\infty, +\infty), \quad t > 0, \]  

\[ u(x, 0, t, \epsilon) = u^0(x), \quad u(x, a, t, \epsilon) = u^1(x), \quad u(x, y, t, \epsilon) = u(x + L, y, t, \epsilon), \quad u(x, y, 0, \epsilon) = u_{init}(x, y), \]  

(157)

Here \( A(u, x, y) = \{A_1(u, x, y), A_2(u, x, y)\}, \epsilon \in (0; \epsilon_0) \) – small parameter. We assume that the functions \( A_i(u, x, y), i = 1, 2, B(u, x, y) \) are smooth enough in \( I_u \times \bar{D} \) where \( I_u \) is allowed interval of \( u \) values, \( \bar{D} = \{(x, y) | \mathbb{R} \times [0, a]\} \), \( u^0(x) \) and \( u^1(x) \) are continuous \( L \)-periodic in \( x \) functions, \( x \in \mathbb{R}, u_{init}(x, y) \) is a continuous \( L \)-periodic in \( x \) function in \( \bar{D} \).

Required conditions

C1. The equation \( \{A(u, x, y), V\} u + B(u, x, y) = 0 \) with the additional condition \( u(x, 0, t, 0) = u^0(x) \) has a solution \( \phi^{(0)}(x, y) \), and with the additional condition \( u(x, a, t, 0) = u^1(x) \) has a solution \( \phi^{(1)}(x, y) \), where \( \phi^{(1)}(x, y) \) are smooth enough in \( \bar{D} \) \( L \)-periodic in \( x \) functions.

Besides the inequality \( \phi^{(0)}(x, y) < \phi^{(1)}(x, y) \) holds when \( (x, y) \in \bar{D} \).

C2. Let the inequalities

\[ A_2(\phi^{(0)}(x, y), x, y) > 0; \quad A_2(\phi^{(1)}(x, y), x, y) < 0 \]  

(158)

hold everywhere in \( \bar{D} \).

We shall investigate the problem solution having a form of a moving front, that is a function close at each moment of time to the surface \( u = \phi^{(0)}(x, y) \) near the border \( y = 0 \) and close to the surface \( u = \phi^{(1)}(x, y) \) near the border \( y = a \) and that undergoes rapid changes from the values on the surface \( u = \phi^{(0)}(x, y) \) to the values on the surface \( u = \phi^{(1)}(x, y) \) in the vicinity of some curve \( y = h(x, t) \). This vicinity is usually called "the inner transition layer".

Let us assume that \( y = h(x, t) \) is a curve on which the solution \( u(x, y, t) \) of the problem at each moment of time has a value equal to half sum of the values \( \phi^{(0)}(x, h(x, t)) \) and \( \phi^{(1)}(x, h(x, t)) \).

Keywords : reaction-diffusion-advection equation; moving front; small parameter; differential inequalities.

2010 Mathematics Subject Classification : 35K57; 34A40.
For the detailed description of the inner transition layer we turn to local coordinates \((l, r)\) with the help of relations
\[
x = l - r \sin \alpha, \quad y = h(l, t) + r \cos \alpha
\]
where \(\sin \alpha = \frac{h_x(l, t)}{\sqrt{1 + h_x^2(l, t)}}\) and \(\cos \alpha = \frac{1}{\sqrt{1 + h_x^2(l, t)}}\). \(\alpha\) is an angle between normal to curve \(y = h(x, t)\) drawn into the region \(y < h(x, t)\) and axis \(y\), \(r\) is the distance from the point in the vicinity of the curve along the normal, \(l\) is the abscissa of a point on this curve, from which the normal is drawn.

C3. Let there exist a smooth curve \(y = h^0(x, t)\) that is defined by the problem
\[
\begin{align*}
\dot{h}_l^0 &= (\varphi^+(x, h^0) - \varphi^-(x, h^0))^{-1} \int_{\varphi^-(x, h^0)}^{\varphi^+(x, h^0)} (h_x^0 A_1(u, x, h^0) - A_2(u, x, h^0)) du, \\
h^0(x, t) &= h^0(x + L, t), \quad h^0(x, t) = h^0(x),
\end{align*}
\]
where \(h^0(x)\) is a curve defined from the equality
\[
u_{init}(x, h^0(x), 0) = \frac{1}{2} (\varphi^-(x, h^0(x), 0) + \varphi^+(x, h^0(x), 0)).
\]

C4. Let the following inequality hold
\[
\int_{\varphi^-(l, h^0(l, t))}^{\varphi^+(l, h^0(l, t))} \left( \frac{h_l}{1 + h_x^2} - A_1(u, l, h(l, t)) \frac{h_x}{\sqrt{1 + h_x^2}} + A_2(u, l, h(l, t)) \frac{1}{\sqrt{1 + h_x^2}} \right) du > 0.
\]
when \(\varphi^-(l, h^0(l, t)) < u < \varphi^+(l, h^0(l, t))\).

Main results
Under the conditions (C1)-(C4) the existence of a solution in a form of a two-dimensional moving front for the problem (1) is proved. An asymptotic approximation in powers of a small parameter of such a solution is constructed. The proof of the existence of a solution approximated by the asymptotics constructed is provided. For the last purpose the asymptotical method of differential inequalities is used.

Acknowledgments
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References
Generalized convolutions for the Hankel transform and related integral operators

Lyubov Britvina

Novgorod State University, Russia

E-mail: lyubov.britvina@novsu.ru

Abstract  The present research is devoted to integral operators related to generalized convolutions (polyconvolutions) for the Hankel transform \((H_\nu f)(x)\). The generalized convolutions defined by the Parseval type equality

\[(H_\nu h_1)(x) = x^{-\nu}(H_\mu f)(x)(H_\mu g)(x), \quad (H_\mu h_2)(x) = x^{-\nu}(H_\nu f)(x)(H_\mu g)(x)\]

are considered in spaces \(L_1(\mathbb{R}_+, \sqrt{t} \, dt)\) and \(L_2(\mathbb{R}_+, t \, dt)\). Existence conditions and boundary properties are found. Watson's type theorems for convolutions are proved. The integral operators with unsymmetrical kernels are studied. Some examples of solving integral equations are given.

Introduction

This research is a continuation of the investigation of convolution operators and their applications to solving convolution equations, given in [1, 2, 3] for the Hankel transform

\[(H_\nu f)(x) = \int_0^\infty f(t) J_\nu(x t) \, t \, dt, \quad x \in \mathbb{R}_+, \]

(162)

where \(J_\nu(z)\) is the Bessel function of the first kind of order \(\nu\), \(\text{Re} \, \nu > -1/2\).

This transform is the most extensively studied area of the theory of Bessel transforms. The Hankel transform is used to solve many problems of mathematical physics. Here we will consider transform (162) in weight Lebesgue spaces \(L_1(\mathbb{R}_+, \sqrt{t} \, dt)\) and \(L_2(\mathbb{R}_+, t \, dt)\) with the norms

\[\|f\|_{L_1(\mathbb{R}_+, \sqrt{t} \, dt)} = \int_0^\infty |f(t)| \sqrt{t} \, dt < \infty, \quad \|f\|_{L_2(\mathbb{R}_+, t \, dt)} = \left(\int_0^\infty |f(t)|^2 t \, dt\right)^{1/2} < \infty.\]

Various generalized convolutions generated by the Hankel transform and other integral transforms can be constructed by using the definition of generalized convolution or polyconvolution introduced by V.A. Kakichev [4, 5]. The corresponding results can be found, for example, in [1, 2, 3, 5].

Let \(A_1, A_2\) and \(A_3\) be linear operators, \(A_j : M_j \rightarrow N_j, \ j = 1, 2\) and \(A_3 : M_3 \rightarrow N_3\). Assume that some weight function \(\alpha(x)\) exists such that for all functions \((A_1 f)(x) \in N_1\) and \((A_2 k)(x) \in N_2\) the product \(\alpha(x)(A_1 f)(x)(A_2 k)(x)\) belongs to the space \(N_3\).

Keywords: Hankel transform; convolution; convolution transform; Watson's theorem; integral equations.

2010 Mathematics Subject Classification: 44A05; 44A35; 45A05; 4SP05.
Definition 8. The generalized convolution, or polyconvolution, of functions \( f(t) \in M_1 \) and \( k(t) \in M_2 \), under \( A_1, A_2, A_3 \), with weight function \( \alpha(x) \), is the function \( h(t) \in M_3 \) denoted by \( \left( f_{A_1} \ast_a k_{A_2} \right)_{A_3}(t) \) for which the factorization property

\[
(A_3 h)(x) = A_3 \left( f_{A_1} \ast_a k_{A_2} \right)_{A_3}(x) = \alpha(x)(A_1 f)(x)(A_2 k)(x)
\]

is valid.

The classical convolution for the Hankel transform was first introduced by Ya.I. Zhitomirskii in 1955. In 1967 V.A. Kakichev [4] constructed this convolution by using Definition 8. A number of convolution constructions involving the Hankel transform was derived by N. X. Thao and N. T. Hai. Some polyconvolutions obtained by the author were exhibited in [1, 2, 3, 6]. The results presented in these research are based on the Kakichev approach to the notion of the polyconvolution.

If one of the functions in the convolution \( \left( f_{A_1} \ast_a k_{A_2} \right)_{A_3}(t) \), say the function \( k(t) \), is fixed, then one can study the transform of convolution type:

\[
A : f \rightarrow \mathcal{L} \left( f_{A_1} \ast_a k_{A_2} \right)_{A_3},
\]

where \( \mathcal{L} \) is an operator. The function \( k(t) \) is called the kernel of the transform \( A \).

Integral transforms related to various convolution constructions was considered by V. K. Tuan, F. Al-Musallam, N. X. Thao, N. T. Hong, S. B. Yakubovich and others. The results by author can be found, for example, in [1, 2, 3, 6].

In this research the generalized convolutions defined by the Parseval type equality

\[
(H_\nu h_1)(x) = x^{-\nu}(H_\mu f)(x)(H_\mu g)(x), \quad (H_\mu h_2)(x) = x^{-\mu}(H_\nu f)(x)(H_\nu g)(x)
\]

are studied in spaces \( L_1(\mathbb{R}^+, \sqrt{t} \, dt) \) and \( L_2(\mathbb{R}^+, td t) \).

Integral operators related to these convolutions are constructed and their existence and boundary properties are found. Also we give some applications to the corresponding class of convolution equations.

References


Numerical simulations of colliding beams dynamics with nonzero crossing angle

Marina A. Boronina\textsuperscript{1}, Ekaterina A. Genrikh\textsuperscript{2}, Vitaly A. Vshivkov\textsuperscript{3}

\textsuperscript{1,2,3}Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, Russia

E-mail: 1boronina@ssd.sscc.ru; 2mesyats@gmail.com; 3vsh@ssd.sscc.ru

Abstract The work is devoted to the problem of charged particle beam dynamics in self-consistent electromagnetic fields in the modern colliders. The colliding beams move in vacuum with highly relativistic speeds ($\gamma \sim 10^3 - 10^5$). The achievement of the high luminosities for the high energy beams finds difficulties due to the beam disruption under the strong electromagnetic fields. The crab-waist scheme [1] of the focusing beam parameters was implemented in physical experiments and has demonstrated its high efficiency for the luminosity gain. We present our first results on the numerical simulations for the beams with the nonzero crossing angle with our parallel fully 3-D algorithm.

Introduction We consider non-stationary problem of relativistic motion of charged particle beams in electromagnetic fields of modern colliders. The solution of the problems can be described with the Vlasov equation for the distribution function of the electrons and positrons

$$\frac{\partial f_{e^+, -}}{\partial t} + \vec{v}_{e^+, -} \frac{\partial f_{e^+, -}}{\partial \vec{r}} + \vec{F}_{e^+, -} \frac{\partial f_{e^+, -}}{\partial \vec{p}} = 0$$

and the set of Maxwell's equations

$$\text{rot} \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$$
$$\text{rot} \vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$
$$\text{div} \vec{E} = 4\pi \rho$$
$$\text{div} \vec{H} = 0$$

where

$$\vec{F}_{e^+, -} = e^+\text{c}^2 (\vec{E} + \frac{1}{c}[\vec{v}_{e^+, -}, \vec{H}])$$

is the Lorentz force, which acts on the particle, $\vec{p}_{e^+, -} = \gamma_{e^+, -} m_e \vec{v}_{e^+, -}$ is the particle moment, $\gamma_{e^+, -} = \sqrt{1 - \frac{\vec{v}_{e^+, -}^2}{c^2}}$ is the relativistic factor of the particle.

In the relativistic particle dynamics the underlying PDEs are highly non-linear, non-stationary and fully 3-D. In the general case the problem can be solved only numerically and places high requirements for the computer resources. We use particle-in-cell method with Langdon-Lazinsky scheme and Boris algorithm in Cartesian coordinate system. We implement special initial and boundary conditions for the electric field to perform simulations with nonzero crossing angle. The question of luminosity enhancement is considered.

Keywords: particle-in-cell methods; beam dynamics; beam-beam effects; crossing angle; self-consistent electromagnetic fields; mathematical modelling; numerical experiments; parallel algorithms; mixed decomposition

2010 Mathematics Subject Classification: 65Z05; 35Q83; 65Y05;
Main results

On the base of our parallel algorithm for 3-D simulations we created the enhanced code for the cases of nonzero crossing angles. We present the numerical results for the colliding beams with Gaussian spatial distributions of cylindrical and ellipsoidal shapes. In the case of cylinder the electromagnetic field of the beams can be calculated analytically and compared with the numerical solution. The luminosity behaviour can be described with analytical formula and its dependence on the crossing angle is connected with the Piwinski angle. The results demonstrate good coincidence with the theoretical values and necessitate next steps on the way of the implementation of the crab-waist scheme in our algorithm.

Acknowledgments

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References


Preliminary results for velocity control of a nanorobot in non-Newtonian fluids using fractional calculus

Clara Ionescu$^{1,2}$, Cristina I. Mureșan$^2$, Isabela R. Birs$^2$, Silviu Folea$^2$

$^1$ Ghent University, Belgium $^2$ Technical University of Cluj-Napoca, Romania

E-mail: $^1$ClaraMihaela.Ionescu@ugent.com; $^2$Cristina.Muresan;Isabela.Birs;Silviu.Folea@aut.utcluj.ro

Abstract Nanomedicine implies nanorobot technology for patient-specific drug delivery systems. But this technology is strongly influenced by the characteristics of the medium in which the nanorobot is operating. In this paper, some preliminary results regarding the velocity control of a nanorobot in non-Newtonian fluids is envisaged.

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References


Keywords: non-Newtonian fluids; drag forces; velocity control; fractional calculus.

2010 Mathematics Subject Classification: 26A33; 34A60; 34G25; 93B05.

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Global exponential stability of delayed inertial neural networks with impulses via inequality techniques

Chaouki Aouiti¹, El abed Assali², Farouk Chérif³

¹,² University of Carthage, Tunisia ³ University of Sousse, Tunisia

E-mail: ¹chaouki.aouiti@fsb.rmu.tn; ²abed.assali@gmail.com; ³ faroukcheriff@yahoo.fr

Abstract This paper analyzes the global exponential stability of a class of delayed inertial neural networks with impulses. The existence of a unique equilibrium point is proved by using contraction mapping principle theorem. Some sufficient stability criteria for the global exponential stability are derived via the Lyapunov functional method and the linear matrix inequality approach by estimating the upper bound of the derivative of Lyapunov functional.

Introduction

We are concerned with the following inertial neural networks

\[ \frac{d^2 x_i(t)}{dt^2} = -a_i \frac{dx_i(t)}{dt} - b_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} d_{ij} f_j(x_j(t - \tau_j(t))) + I_i \] (163)

for \( i = 1, 2, \cdots, n \) where the second derivative is called an inertial term of system (163); \( x_i(t) \) denote the state of the \( i^{th} \) neuron at time \( t \); \( a_i, b_i > 0 \) are constants; \( c_{ij} \) and \( d_{ij} \) are connection weights related to the neurons without delays and with delays, respectively; \( f_j(.) \) denote the activation function of \( j^{th} \) neuron at time \( t \), \( j = 1, 2, \cdots, n \); \( \tau_j(t) \) donate the time-varying delay of \( j^{th} \) neuron at time \( t \) which satisfies \( 0 \leq \tau_j(t) \leq \tau \) and \( \dot{\tau}_j(t) \leq \rho \) \( \leq 1 \); \( \tau \) and \( \rho \) are constant; \( I_i \) is the external input on the \( i^{th} \) neuron.

The initial conditions of inertial neural networks (163) are

\[ x_i(s) = \varphi_i(s), \quad \frac{dx_i(s)}{dt} = \psi_i(s), \quad s \in [-\tau, 0], \quad i = 1, 2, \cdots, n \] (164)

where \( \varphi_i(.) \) and \( \psi_i(.) \) are real-valued continuous functions on \([-\tau, 0]\).

We make the following assumption: \( \forall u, y \in \mathbb{R}: \)

(111) \( |f_i(u + y) - f_i(u)| \leq L_i|y|, f_i(0) = 0, \quad i = 1, 2, \cdots, n. \)

Main results

We obtain existence, uniqueness and global exponential stability of delayed inertial neural networks with impulses. Let \( B = \text{diag}(b_1, b_2, \cdots, b_n), C = (c_{ij})_{n \times n}, H = \text{diag}(1 + h_1^{(1)}(1 + h_2^{(1)}(1 + h_3^{(1)}(1 + h_4^{(1)}(1 + h_5^{(1)}))))) \), \( \overline{H} = \text{diag}(1 + \overline{h}_1^{(1)}(1 + \overline{h}_2^{(1)}(1 + \overline{h}_3^{(1)}(1 + \overline{h}_4^{(1)}(1 + \overline{h}_5^{(1)}))))) \)

Keywords: Inertial neural networks; Impulse; Time varying delay; Global exponential stability.

2010 Mathematics Subject Classification: 92B20; 34D23.

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Theorem 44. Suppose that the system \((163)\) satisfies (H1) and suppose that:

\((H2): \ b_j > \sum_{i=1}^{n} (|c_{ij}| + |d_{ij}|) L_j, \ j = 1, 2, \cdots, n.\)

Then the system \((163)\) has a unique equilibrium point \(x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T \in \mathbb{R}^n.\)

Theorem 45. Under the conditions in theorem 44, assume that there exist constants \(\epsilon > 0, \sigma > 0\) and \(n \times n\) symmetric definite positive matrices \(P\) and \(Q\) which satisfy:

(i) \(\sigma \lambda_{\text{max}}(Q) < e^{-\epsilon t} \min_{1 \leq j \leq n} \{ \frac{1}{b_j} \},\)

(ii) \(\bar{\epsilon} + 1 + \max_{1 \leq i \leq n} \{ \frac{1}{b_i} \} + \lambda_{\text{max}}(P) \max_{1 \leq j \leq n} \{ L_j^2 \} + \frac{1}{1 - \rho} \max_{1 \leq j \leq n} \{ \frac{L_j^2}{b_j} \} < 2 \min_{1 \leq j \leq n} \{ \xi_j \},\)

(iii) \(1 + \bar{\epsilon} \max_{1 \leq i \leq n} \{ \frac{1}{b_i} \} + \max_{1 \leq i \leq n} \{ \frac{\alpha_i^2}{b_i} \} + \lambda_{\text{max}}(B^{-1} CP^{-1} C^T B^{-1}) + \frac{1}{\sigma} \lambda_{\text{max}}(B^{-1} DQ^{-1} D^T B^{-1}) < 2 \min_{1 \leq j \leq n} \{ \frac{L_j^2}{b_j} \},\)

(iv) there exist constants \(\nu \geq 0, \bar{\alpha} \in [0, \bar{\epsilon}]\) such that:

\[ \sum_{k=1}^{m} \ln \max \{ \chi_k, \frac{\chi_k}{\lambda_{\text{min}}(B^{-1})} \} < \nu + \bar{\alpha}(t_m - t_0), \ \forall \ m \in \mathbb{Z}_+, \text{ where } \chi_k \text{ is the largest eigenvalue of } H_k.\]

Then the equilibrium point of system \((163)\) is globally exponentially stable and the approximate exponentially convergent rate is \((\epsilon - \bar{\alpha})\).

References


Near-perfect matchings on $P_m \times P_n$ graphs of odd order

S. N. Perepechko

Petrozavodsk State University, Russia

E-mail: persn@newmail.ru

Abstract  A set of linear recurrence relations and generating functions is obtained for the number of near-perfect matchings on the graphs $P_m \times P_n$ for fixed odd values of the parameter $3 \leq m \leq 15$. The coefficients of the asymptotic expansions are calculated for $3 \leq m \leq 21$. A comparison is made with the results of numerical calculations performed earlier by Kong.

Introduction

We consider the problem of counting near-perfect matchings on rectangular lattices $P_m \times P_n$ when parameters $m$ and $n$ are both odd. A distinctive feature of these matchings is the presence of exactly one node which is left unmatched. This node will be called vacancy.

This combinatorial problem is closely related to the dimer problem – the well-known lattice model of statistical physics. Due to a highly specific structure of graphs, popular in physical applications, maximum matchings, as a rule, correspond to perfect matchings in the case when the order of the graph is even. But if the order of the graph is odd, then maximum matchings are often near-perfect matchings. When activities of all dimers are the same, generating function for the number of maximum matchings is identical to the partition function in the dimer model.

Although closed-form expressions for the number of perfect matchings on $P_m \times P_n$ graphs have long been known, similar formulas for near-perfect matchings were obtained only under very restrictive assumptions. Traditional approaches, based on the extended Temperley bijection [1] or on direct Pfaffian evaluation, are successful only in cases where the vacancy is on the boundary. Summation over all vacancy locations leads to laborious calculations due to the exponential growth of the order of transfer matrix [2].

Let us denote by $K_{m,n}^N$ the number of near-perfect matchings on $P_m \times P_{2n+1}$ graph and by $K_{m,n}^P$ the number of perfect matchings on $P_m \times P_{2n}$ graph. Here and henceforth $m$ will be assumed to be odd.

The results of the exact calculation of $K_{m,n}^N$ for fixed $3 \leq m \leq 19$ revealed a different behavior of the free energy per node in comparison with the even values of $m$ [2]. Moreover, numerical fitting of the available data clearly indicated the presence of a logarithmic correction in the asymptotic expansion of $K_{m,n}^N$. In contrast to [1], this correction was manifested even for small $m$.

The most interesting part of [2] is the estimate of $a_k$ by fitting $\ln(K_{m,n}^N)$ for $n \to \infty$ with the following function:

$$\frac{\ln(K_{m,n}^N)}{mn} = \sum_{k=0}^{4} \frac{a_k}{n^k} + \frac{h}{m} \frac{\ln(n + 1)}{n}.$$  (165)

It turned out that $h = 1$ for all $m$ studied in [2].

Keywords: near-perfect matchings; generating functions; dimer problem; free energy.

2010 Mathematics Subject Classification: 05C30; 82B20; 65B05.
Main results

We recall one useful property of the sequence \( \{K_{m,n}^P\} \) for fixed \( m \). This sequence satisfies a linear recurrence relation, and its generating function \( G^P_m(z) = \sum_{n=0}^{\infty} K_{m,n}^P z^n \) is rational. At the same time, it follows from the existence of the transfer matrix and the Cayley-Hamilton theorem that the sequence \( \{K_{m,n}^N\} \) will have the same property and its generating function \( G^N_m(z) = \sum_{n=0}^{\infty} K_{m,n}^N z^n \) is also rational.

In this case, both the recurrence relation itself and the generating function \( G^N_m(z) \) can be reconstructed from a sufficiently long initial segment \( \{K_{m,n}\} \). A few simple examples are given below:

\[
G^N_1(z) = \frac{1 + z - z^2}{(1 - 4z + z^2)^3}, \quad G^N_2(z) = \frac{3z^2 + 18z^3 + 45z^4}{(1 - 9z + 18z^2 - 18z^3 + z^4)^3}.
\]

Calculation of the values of \( K_{m,n}^N \) and the derivation of recurrence relations were carried out in a 64-bit version of the computer algebra system Maple. The available computational facilities allowed us to find closed-form expressions for \( G^N_m(z) \) and calculate the exact values of \( a_k \) in (165) for \( 3 \leq m \leq 15 \). In the particular case \( m = 3 \), these coefficients are given by the following expressions:

\[
a_0 = \frac{1}{6} \ln(2 + \sqrt{3}) \approx 0.219493, \quad a_1 = a_0 + \frac{1}{3} \ln\left(\frac{1 + 2\sqrt{3}}{12}\right) \approx -0.110120, \quad a_2 = \frac{4}{99} (\sqrt{3} - 6) \approx -0.172442, \\
a_3 = \frac{4}{1089} (40 - 3\sqrt{3}) \approx 0.127838, \quad a_4 = \frac{4}{107811} (97\sqrt{3} - 2826) \approx -0.0986167.
\]

Comparison of our results with the data of Table II [2] showed that even for \( m = 3 \) only the first three coefficients obtained by Kong have the declared accuracy. The coefficient \( a_3 \) is calculated with 4 correct significant digits, and \( a_4 \) – with only one. With an increase in \( m \), the error in the data in Table II also increases.

The study of generating functions \( G^N_m(z) \) made it possible to establish a close connection between their denominators and the denominators \( G^P_m(z) \). As it seems to us, the observed regularity will occur for all odd \( m \), however, due to the lack of formal proof, we formulate the result in the form of a conjecture.

Conjecture 1. For all odd values of \( m \) denominator \( G^N_m(z) \) is always the square of denominator \( G^P_m(z) \).

The validity of conjecture 1 means that for fixed \( m \) the asymptotic \( K_{m,n}^N \) has the form

\[
K_{m,n}^N \sim (\alpha_n + \beta_m) \Lambda(m) z^n, \quad \Lambda(m) = \prod_{j=1}^{[m/2]} \left[ \cos\left(\frac{\pi j}{m+1}\right) + \sqrt{1 + \cos^2\left(\frac{\pi j}{m+1}\right)} \right].
\]

However, in this case only 2 coefficients in (165) are really independent. In fact, all \( a_k \) (\( k > 0 \)) can be expressed through \( \alpha_m \) and \( \beta_m \).

A numerical estimate of the coefficients \( \alpha_m \) and \( \beta_m \) was obtained for \( m = 17, 19, 21 \). The calculations were carried out in two stages. At the first stage, a sequence of relations \( \{K_{m,n}^N, K_{m,n+1}^N\} \) was constructed. Due to the logarithmic convergence, in the second stage, the acceleration method based on the use of Neville tables was applied to this sequence. The accuracy of the estimates obtained by such an approach is higher than in [2].

References


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Simulations for efficient combination of two lower bound functions in univariate global optimization

Mohammed Chebbah\(^1\), Mohand Ouanes\(^2\), Ahmed Zidna\(^3\)

\(^1\)TIZI OUZOU University, Algeria, \(^2\)TIZI OUZOU University, Algeria, \(^3\)University of Metz, France

E-mail: \(^1\)chbbhea@yahoo.fr; \(^2\)ouanes.mohand@yahoo.fr; \(^3\)ahmed.zidna@univ-lorraine.fr

Abstract We propose a new method for solving univariate global optimization problems by combining a lower bound function given in αBB method \([1]\), with the improved lower bound function of the method developed in \([5]\). The new lower bound function is better than the two lower bound functions by its construction. The complementarity of the two lower bound functions allows us to derive the convex/concave test and the pruning step which accelerate the convergence of the proposed method. Illustrative examples are treated efficiently.

References


Keywords: Global optimization, αBB method, quadratic lower bound function, Branch and Bound, pruning method.

2010 Mathematics Subject Classification: 34K60; 46N10.
Abstract  Fractional velocity is defined as the limit of the fractional difference quotient if it exists. This contribution demonstrates the use of fractional velocity to develop fractional Taylor expansions. Explicit formulas for the coefficients are presented for three classes of functions.

References


Keywords: Hölder functions; analytic functions; power series expansions

2010 Mathematics Subject Classification: 26A27; 26A16; 4104
Slow strain waves in blocky geomedium. Mathematical modelling and numerical estimations

Artyom Gerus\textsuperscript{1,2}, Alexander Vikulin\textsuperscript{1}

\textsuperscript{1}Institute of Volcanology and Seismology FEB RAS, Russia \quad \textsuperscript{2}Vitus Bering Kamchatka State University, Russia

E-mail: gerus@kscnet.ru

Abstract We introduce one possible approach to model slow geodynamic waves with sine-Gordon equation. It is based on a hypothesis about blocky structure of the Earth's crust. We show some numerical results of our modelling which are consistent with the real data.

Introduction

The idea of stress propagation by means of slow geodynamic waves goes back to the discovery of earthquake migration, which was made by Richter in the middle of the last century. The main problem of this concept is to directly detect existence of such waves with some instruments: it is impossible because of very low amplitude and propagation velocity. The only possible way is to detect these waves indirectly, observing variations of some geophysical fields, such as variations in subsoil radon concentration, acoustic emission intensity or GPS stations position.

To model such kind of waves, we use an approach which was introduced by A.V. Vikulin; it is called "A rotational model of block geomedium" [1]. We consider that the crust along the geodynamically active zones of the planet consists of blocks. The term "block" usually means a rigid and non-deformable volume of material which is associated with earthquake focal area. The idea is the following: when a single block moves with the whole crust (turns by an angle $\beta$, this angle is visible from the center of the planet), its intrinsic angular momentum $M$ changes the direction, but due to the law of conservation the moment $K$ appears. This moment of force is responsible for the appearance of elastic stress field. During the translational motion of the block these stresses are accumulating in the crust, and this explains the phenomenon of 'energy-saturation'.

A cooperative movement of blocks in a block chain is modelled with a well-known sine-Gordon equation:

$$\frac{\partial^2 \theta}{\partial \xi^2} - \frac{\partial^2 \theta}{\partial \eta^2} = \sin \theta. \quad (166)$$

It has exact solutions in a form of solitary waves which fit the whole theory very well.

To model a real process by this classic SG equation (166), we have to add some terms that will correspond to the inhomogeneity of real crust movements. A modified and more accurate equation is the following:

$$\frac{\partial^2 \theta}{\partial \xi^2} - \frac{\partial^2 \theta}{\partial \eta^2} = \sin \theta + \alpha \frac{\partial \theta}{\partial \eta} + \mu \delta(\xi) \sin \theta. \quad (167)$$

It has no exact analytical solutions, so we have to use numerical methods to investigate dynamical parameters of its wave solution.

Keywords: strain waves; blocky geomedium; sine-Gordon equation; solitons; seismic process; earthquakes migration.

2010 Mathematics Subject Classification: 35Q51; 70K70; 74J35; 86A17.
Main results

The calculated parameters of our model were the following: a position of the wave along the block chain $X$, velocity of the wave $U$ and rotation velocity $\dot{\theta}$. The last one is just a speed of displacement of blocks in a chain. We plotted these parameters for different combinations of the two key values: the amount of friction in a chain ($\alpha$) and the magnitude of wave motion inhomogeneity ($\mu$) (see [167]). One of the most interesting things about these plots is the fact that ratios between two maximums of $\dot{\theta}$-plot and between their durations agree with ratios between total seismic energies of foreshock and aftershock stages and durations of these stages [?]. Also the terminal velocity of our model wave is of the same order of magnitude as estimated velocity of geodynamic waves including migration waves. So we can consider our model as consistent with the real data on seismic process.

References


Symmetry properties of two-phase filtration model with space-fractional derivatives

Rafail K. Gazizov, Alexey A. Kasatkin, Stanislav Yu. Lukashchuk

Ufa State Aviaton Technical University, Russia

E-mail: gazizovrk@gmail.com; alexei_kasatkin@mail.ru; lsu@ugatu.su

Abstract The process of capillary counter-current imbibition is considered. We use modified Darcy's law that contains space-fractional derivatives corresponding to non-local effects in filtration. The resulting nonlinear anomalous diffusion-type equation is analyzed. Lie point symmetries are found and used to construct the group-invariant solutions.

Acknowledgments

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References


Keywords: Lie symmetries; fractional derivatives; two-phase filtration

2010 Mathematics Subject Classification: 26A33; 35B06; 76S05.
Analysis and resolving of singularly perturbed elliptic Dirichlet problem with three-band boundary layer

V.A. Beloshapko

1 Faculty of Physics, Lomonosov Moscow State University, Leninskie Gory, Moscow, 119991 Russia.

E-mail: postvab@rambler.ru

Abstract  Singularly perturbed elliptic Dirichlet problem in the case of degenerate equation’s multiply root is researched. The complete asymptotic expansion of the problem’s solution is considered and proved. It has some essential features in compare to the problem of degenerate equation’s simple root. Regular and boundary parts of asymptotic expansion are series in fractional powers of small parameter. The boundary layer has three zones. There are several boundary-layer variables of different scale.

Acknowledgments

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References


Keywords: singularly perturbed problem, case of the degenerate equation’s multiple root, three-zone boundary layer, solution’s asymptotic expansion.

2010 Mathematics Subject Classification: 26A33; 34A60; 34G25; 93B05.
Strong Stability of the Embedded Markov Chain In an PH/M/1 Queue

Yasmina Djabali¹, Boualem Rabta², Djamil Aissani³

¹²³ Research Unit LaMOS (Modeling and Optimization of Systems)
Bejaia University, Algeria

E-mail: ¹dj.mina06@yahoo.fr; ²brabta@yahoo.fr; ³lamos_bejaia@hotmail.com

Abstract The main objective of this paper is to provide by means of the strong stability method, the mathematical justification of the approximation method by phase-type distributions. We are interested in the approximation of a class of queueing systems through the replacement of their inter-arrival distributions by phase type distributions. The approximating distribution should be close to the original one in some sense. However, considering the calculations’ complexity as well as the algorithm’s efficiency requirements, it is not possible to exactly match the two distributions. Additionally, the model’s parameters are estimated by means of statistical methods which constitutes another source of perturbations. Hence, it is important to check the robustness of the system and estimate the resulting deviation of its characteristics. The strong stability method is a powerful approach for this purpose. We consider the approximation of GI/M/1 queueing systems by PH/M/1 systems, where PH refers to a hyper-exponential H₂ or a hypo-exponential HOE₂ distribution depending on the value of the coefficient of variation of the original inter-arrival distribution. We provide the qualitative confirmation of the stability as well as quantitative estimates of the approximation error in both cases.

References


Keywords: Queueing systems; phase-type distributions; Perturbation; Strong stability; Quantitative estimates.
2010 Mathematics Subject Classification: 60K25; 03C45.
On the Dynamics of a Viral Marketing Model with Optimal Control using Indirect and Direct Methods

João N.C. Gonçalves¹, Helena Sofia Rodrigues²,³, M. Teresa T. Monteiro¹,⁴

¹ Algoritmi R&D Center, University of Minho, Braga, Portugal
² Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro
³ School of Business Studies, Polytechnic Institute of Viana do Castelo, Valena, Portugal
⁴ Department of Production and Systems, University of Minho, Braga, Portugal

E-mail: ¹ jncostagoncalves@gmail.com; ²,³ sofiarodrigues@esce.ipvc.pt; ¹,⁴ tm@dps.uminho.pt

Abstract  The complexity of optimal control problems requires the use of numerical methods to compute control and optimal state trajectories for a dynamical system, with the aim of optimize a particular performance index. Based on a real viral advertisement, this article studies the dynamics of a SIR viral marketing epidemic model with optimal control under the implementation of indirect and direct methods. In order to maximize the spread of information with a low cost, an optimal control problem is formulated and studied in the light of indirect and direct methods. The existence and uniqueness of the solution are proved. Our results suggest that low investment costs in marketing strategies fulfill the proposed trade-off without compromising the financial capacity of a company. Numerical simulations also show that the cost of implement control policies is a crucial parameter for the spreading of marketing messages, by marketing professionals, within a target population.

Acknowledgments

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References


Keywords: indirect methods; direct methods; optimal control; viral marketing; SIR epidemiological model.
2010 Mathematics Subject Classification: 34H05; 49J15; 49K15; 91F99.
Numerical Modeling of the Generation of the Electromagnetic Radiation by the Beam-Plasma Interaction with Using of the Particle-in-Cell Method

Anna Efimova¹, Evgeny Berendeev², Galina Dudnikova³

¹,² Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Russia
³ Institute of Computational Technologies SB RAS, Novosibirsk, Russia

E-mail: ¹efimova@ssd.sscc.ru, ²berendeev@ssd.sscc.ru, ³dudn@ict.nsc.ru

Abstract The work is devoted to computer modeling of the electromagnetic radiation generated in the electron beam-plasma system with using particle-in-cell (PIC) method. In 2D3V model electron beam entering into the plasma along magnetic field lines through one boundary and leaving it through the other is a producer of continuous pumping of plasma oscillations. The corresponding numerical model and parallel algorithm and code has been developed. The dominant harmonics of the electromagnetic fields in the vacuum have been found.

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References


Keywords: particle-in-cell methods; Maxwell's equations; ABC boundary condition; Mur's boundary condition; Vlasov equation; generation of electromagnetic radiation

2010 Mathematics Subject Classification: 82D10; 33F05; 33D67; 65D15; 65Y05; 78A25
Preliminary and shrinkage estimation in quantile regression model with iid errors

Samil Sik\textsuperscript{1}, Bahadir Yüzbasi\textsuperscript{2}, Yasin Asar\textsuperscript{3}

\textsuperscript{1,2}Inonu University, Turkey \textsuperscript{3}Necmettin Erbakan University, Turkey

E-mail: \textsuperscript{1}mhmd.sml85@gmail.com; \textsuperscript{2}b.yzb@hotmail.com; \textsuperscript{3}yasinasar@hotmail.com

Abstract In this study, we consider preliminary test and shrinkage estimation strategies for quantile regression models. In classical Ordinary Least Squares (OLS) method, the relationship between the explanatory and explained variables in the coordinate plane is estimated with a mean regression line. In order to use OLS estimator, there are three assumptions on the error terms showing white noise process of the regression model, also known as Gauss-Markov Assumptions, must be met: (1) The error terms have zero mean, (2) The variance of the error terms is constant and (3) The covariance between the errors is zero i.e., there is no autocorrelation. However, data in many areas, including econometrics, survival analysis and ecology, etc. does not provide these assumptions. First introduced by Koenker, quantile regression has been used to complement this deficiency of classical regression analysis and to improve the least square estimation. The aim of this study is to improve the performance of quantile regression estimators by using pre-test and shrinkage strategies. A Monte Carlo simulation study including a comparison with $L_1$ type estimators such as Lasso, Ridge and Elastic Net are designed to evaluate the performances of the estimators. A real data example is given for illustrative purposes. Finally, we obtain the asymptotic results of suggested estimators.

References


Keywords: Shrinkage estimation; Penalty Estimation; Robust Estimation; Multiple Regression Model.

2010 Mathematics Subject Classification: 62G05; 62G35.
Shrinkage and penalty estimation in quantile regression model with autoregressive errors

Ahmet Demiralp, Yasin Asar, Bahadir Yüzbasi

1,3 Inonu University, Turkey 2 Necmettin Erbakan University, Turkey

E-mail: ahmt.dmrlp@gmail.com; yasinasar@hotmail.com; b.yzb@hotmail.com;

Abstract In a classical regression model, it is usually assumed that the explanatory variables are independent of each other and error terms are normally distributed. But when these assumptions are not met, situations like the error terms are not independent or they are not identically distributed or both of these, LSE will not robust. Hence, quantile regression has been used to complement this deficiency of classical regression analysis and to improve the least square estimation (LSE). In this study, we consider preliminary test and shrinkage estimation strategies for quantile regression models with non-iid errors. A Monte Carlo simulation study is conducted to assess the relative performance of the estimators. Also, we numerically compare their performance with Ridge, Lasso, Elastic Net penalty estimation strategies. A real data example is presented to illustrate the usefulness of the suggested methods. Finally, we obtain the asymptotic results of suggested estimators.

References


Keywords: Shrinkage estimation; penalty estimation; robust estimation; multiple regression model.
2010 Mathematics Subject Classification: 62G05; 34K60.
Solution of ill-posed problems by regularized double period method on the example of Fredholm equation

Aleksandr A. Belov\textsuperscript{1}, Igor A. Fedorov\textsuperscript{2}

Lomonosov Moscow State University, Russia

E-mail: \textsuperscript{1}aa.belov@physics.msu.ru; \textsuperscript{2}igorfedorov0997@gmail.com

Abstract Ill-posed problems are proposed to be solved by regularized double period method. The solution is represented as Fourier series of the main period harmonics with addition of several doubled period harmonics providing good approximation near the boundaries and extrapolation properties. For regularization, A. N. Tikhonov stabilizer of high even order is implied. It allows to avoid solution distortion and damps non-physical oscillations on the solution curve.

Problem

Many applied problems lead to solution of operator equation $Au = f$. Let us consider it on the example of Fredholm equation

$$\left( Au(t) \right)(t) = \int_a^b K(t,s)u(s)ds = f(t), \quad t \in [c,d]. \tag{168}$$

This problem is ill-posed and for its solving regularization is implied. There exist several regularization techniques. The most common and universal one consists in adding A. N Tikhonov stabilizer to the residual and solving the following minimization problem \cite{1}:

$$\int_c^d \left( (Au(t) - f(t))^2 + \sum_{l=0}^{L} w_l(u) \right) dt \rightarrow \min, \quad w_l(u) = \alpha_l \int_a^b \left( u^{(l)}(t) \right)^2 ds. \tag{169}$$

The problem \cite{169} is commonly reduced to boundary value problem (BVP) for variation Euler equation. Its order is $2L$, i.e. twice as high as the derivative order $L$ used in the stabilizer. This traditional approach encounters following difficulties.

Firstly, the BVP requires introduction of $2L$ boundary conditions (BCs). The BCs following from the stabilizer lead to sufficient distortion of the solution near the boundaries. For example, if the stabilizer includes only $w_1$, the corresponding BCs take the form $u(a) = u(b) = 0$ which is often unnatural.

Secondly, the numerical solution of the BVP is a considerable difficulty itself due to ill-conditionality of the problem and very large round-off errors. For example, on a mesh with $N = 1000$ nodes, the round-off errors consume $\sim 5$ decimal digits of the solution for $L = 1$ and even $\sim 10$ for $L = 2$. Thereby, traditionally, low order stabilizers $w_0$ and $w_1$ are implied. But they considerably distort the form of the solution curve and its high slope regions.

Keywords: ill-posed problems, regularization, Tikhonov stabilizer, double period method

2010 Mathematics Subject Classification: 65J20, 45B05, 42A16
Regularized double period method (RMDP)

In the present work, we propose solving such problems with regularized double period method [2]. This approach allows to overcome the mentioned difficulties. The main idea is as follows.

The solution is represented as a specific overdetermined Fourier series consisting of \( N \gg 1 \) of sine-cosine pairs corresponding to the main period \([a, b]\) and \( M \sim 1/5 \) odd harmonics of the doubled period [3]

\[
\varphi_n(x) = \begin{cases} 1, & 0 \leq n \leq 2N \\ \cos x/2, & 2N + 1 \leq n \leq 2N + M \\ \sin x/2, & \ldots \end{cases}
\]

Fourier coefficients \( a_n \) are determined via least-squares method. These series provide good approximation not only inside the \([a, b]\) segment, but also near its boundaries and even allow some extrapolation beyond them.

For regularization, \( w_2(u) \) stabilizer is implied without lower order terms \( w_0(u) \) and \( w_1(u) \). This permits to avoid solution curve distortion both in high slope regions and near the segment boundaries. Simultaneously, the stabilizer damps high-frequency non-physical oscillations of the solution curve providing its physically correct behavior.

The \( w_2 \) stabilizer may affect only high curvature parts of the solution (e.g., narrow resonances). In order to prevent this, stabilizer \( w_4(u) \) can easily be introduced.

For Fourier series (170), both \( w_2 \) and \( w_4 \) can be calculated explicitly, so the minimization problem (169) is reduced to a simple least-squares system instead of complicated BVP

\[
\sum_i \frac{1}{\delta_i^2} \left( \sum_n a_n \varphi_n(t_i) - f(t_i) \right)^2 + w_2 \left( \sum_n a_n \varphi_n \right) \rightarrow \min.
\]

Here, \( t_i \) is a discrete mesh in \([c, d]\) and \( \delta_i \) are weights corresponding to these points. Introduction of the stabilizers sufficiently improves the conditionality of the system [173]. The proposed approach can be applied in similar manner to a wide range of ill-posed problems.

Acknowledgments

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References


Optimization of an Isotropic Metasurface on a Substrate

Zhanna O. Dombrovskaya¹, Alexander N. Bogolyubov²

M.V. Lomonosov Moscow State University, Russia

E-mail: ¹dombrovskaya@physics.msu.ru; ²bogan7@yandex.ru

Abstract Basing on the idea of regularization, we propose an actual well-posed formulation of the problem for accurate optimization of all-dielectric isotropic substrated metasurface parameters at normal incidence. The limitations caused by the specifics of the technological process and the applicability of the physical model are taken into account. The behavior of the constructed functional is discussed.

Introduction

In this paper, we consider an isotropic metafilm with period $p$ ($2a < p \ll \lambda_0$), composed of spherical dielectric scatterers with radius $a$ and permittivity $\tilde{\varepsilon} = \varepsilon' + i\varepsilon''$ located at “air-dielectric” interface (permittivity $\varepsilon$). Such structures are widely used as light-trapping covers, in the Raman spectroscopy and other applications. Depending on the particular task, the metasurface is required to provide maximum or minimum values of the reflection, transmission or absorption coefficients. Our goal is to design the parameters $p, a$ and $\tilde{\varepsilon}$ providing a priori given dependencies $|r(\lambda, \varepsilon)|^2$ and $|t(\lambda, \varepsilon)|^2$. This problem is ill-posed [1].

Direct and inverse problem

In contrast to three-dimensional metamaterial, metasurface is described by electric $\chi_{es}$ and magnetic $\chi_{ms}$ Surface susceptibilities [2] and not by means of bulk material parameters of homogeneous layer [3]. In the case of normal incidence, the structure under consideration is characterized by the following reflection $R$ and transmission $T$ coefficients (temporal dependence $e^{i\omega t}$) [4]:

$$R(\lambda, p, a, \tilde{\varepsilon}, \varepsilon) = \frac{(1 + e)(1 - \sqrt{\varepsilon}m) - (\sqrt{\varepsilon} - e)(1 + m)}{(1 - e)(1 - \sqrt{\varepsilon}m) + (e - \varepsilon)(1 - m)}, \quad T(\lambda, p, a, \tilde{\varepsilon}, \varepsilon) = \frac{(1 + e)(1 + m) + (1 - e)(1 + m)}{(1 - e)(1 - \sqrt{\varepsilon}m) + (e - \varepsilon)(1 - m)},$$

where $e = ik\chi_{es}/2$, $m = -ik\chi_{ms}/2$; $k = 2\pi/\lambda$ is the wave number.

We introduce a vector $x = \{p, a, \varepsilon', \varepsilon''\}$, describing the optimization parameters of the metafilm. Let $f(\lambda, \varepsilon) = |r(\lambda, \varepsilon)|^2$ be the reflectivity of the metasurface given on the interval $[\lambda_1, \lambda_2]$ for some dielectric substrate. In general, it is a given function from $L_2$ space, but commonly in applications it is often required to achieve maximum $f(\lambda, \varepsilon) \equiv 1$ (a perfect mirror) or minimum $f(\lambda, \varepsilon) \equiv 0$ (an anti-reflective coating) reflection. Let $E_4$ be a 4-dimensional vector space, $C_4$ is a closed convex set

$$C_4 = \{x \in E_4 \colon \; p > 2a, \; a > 0, \; \varepsilon'_{\text{min}} \leq \varepsilon' \leq \varepsilon'_{\text{max}}, \; \varepsilon'' \geq 0\}.$$

The problem is to approximate the substrated metasurface reflectivity $R(\lambda, p, a, \tilde{\varepsilon}, \varepsilon)$ provided by nonlinear operator $A(x, \lambda, \varepsilon) = |R(\lambda, p, a, \tilde{\varepsilon}, \varepsilon)|^2$, which is described by $x$-vector at $C_4$, to the required $f(\lambda, \varepsilon)$ within some accuracy $\delta$.

Keywords: optimization; substrated metasurfaces; regularization
2010 Mathematics Subject Classification: 78M50.
taking into account additional restrictions on the synthesized structure. Such a restriction may consist in obtaining the largest possible size of meta-atoms since this significantly simplifies the fabrication of identical samples. We emphasize that formulas (174) are valid only in the case of a dipole approximation. The replacement of sphere by the pair of electric and magnetic dipoles is valid for samples with \( a \leq \lambda_0/s \), where \( s \) is constant (i.e., for silica glass \( s = 6 \), see [5]), \( \lambda_0 \) is the wavelength of the incident monochromatic wave.

Then the inverse problem can be formulated as follows. It is required to determine the vector \( \mathbf{x} \), which minimizes the functional

\[
F^\beta[\mathbf{x}] = \| A(\mathbf{x}, \lambda, \varepsilon) - f(\lambda, \varepsilon) \|^2_{L^2} + \beta |a - \lambda_0/s| = \min, \quad \mathbf{x} \in C_4, \tag{176}
\]

where \( \beta > 0 \) is the regularization parameter [1], which should be chosen according to \( \delta \). From (176) we can find \( \mathbf{x} \) for different values of \( \beta \). Note that the operator \( A(\mathbf{x}, \lambda, \varepsilon) \) and the input data \( f(\lambda, \varepsilon) \) are supposed to be given exactly, so the statement (176) does not take their errors into account. However, it is suitable for optimizing the geometric and material parameters of the metasurface in order to achieve the desired properties.

For isotropic substrated metafilm, operator \( A(\mathbf{x}, \lambda, \varepsilon) \) is given by the analytical formula from (174), therefore, for the minimization of the functional (176), we can imply numerical methods requiring its derivatives. Preliminary analysis showed that \( F^\beta[\mathbf{x}] \) has a large number of local minimums. In order to find all of them, it is recommended to choose \( \sim 10^4 \) pseudorandom points from \( C_4 \) as initial estimates. From these minimums we choose the best one which meets additional physical and technical requirements: dielectric permittivity \( \varepsilon \) should correspond to chemically inert substance in a solid state, meta-atoms should not be fragile, the metafilm should possess good adhesive properties in relation to the substrate, etc. The obtained minimum is not necessarily global.

Acknowledgments

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References


Numerical approximation of tempered fractional terminal value problems

Magda Rebelo\textsuperscript{1}, Maria Luísa Morgado\textsuperscript{2}

\textsuperscript{1} CMA and Department of Mathematics, Universidade NOVA de Lisboa, Portugal
\textsuperscript{2} CMAT- Pole CMAT-UTAD and Department of Mathematics, University of Trás-os-Montes e Alto Douro, Portugal.

E-mail: \textsuperscript{1} msjr@fct.unl.pt; \textsuperscript{2} luisam@utad.pt

Abstract In this work we consider a class of tempered fractional terminal value problems of the Caputo type. We present some results of existence and uniqueness of the solution and suggest a numerical scheme for the approximation of such problems. Some numerical examples are considered in order to illustrate the efficiency of the proposed numerical method.

Acknowledgments

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References


Keywords: Tempered fractional derivatives; Caputo derivative; terminal value problems; numerical methods; shooting method.

2010 Mathematics Subject Classification: 65L05; 34A08.
Modeling of Information Warfare in a Society

A.P. Mikhailov, A.P. Petrov, O.G. Proncheva

1,2,3 Keldysh Institute of Applied Mathematics, Russia; 3 Moscow Institute of Physics and Technology, Russia

E-mail: 1 apmikhailov@yandex.ru; 2 petrov.alexander.p@yandex.ru; 3 olga.proncheva@gmail.com

Abstract Mathematical models of information warfare in a society are considered. We focus on basic models of information attack and information warfare extended to include several additional factors. Also the model built on Rashevsky’s neurological scheme are considered.

Introduction

The basic model of information attack [1] builds on the following assumptions. An information source broadcasts to a large group of individuals. An individual who is ignorant of the information can receive it either from the media or a previously informed individual (spreader). It was shown [2] that the number of individuals is a monotonically increasing function of time. Expansion of the score of the topic was made in several ways. Three additional factors of information dissemination were included in [3], the model of information warfare was constructed in [4], In [5] the case when one of competitors periodically destabilizes the system with short abrupt increasing of intensity of media propagandawas considered in [4]. Finally, the model built on Rashevsky’s neurological scheme [6] was constructed in [7]. All the results are used in this research.

Description of the model

Information warfare arises when social community is potentially exposed two (or more) competitive information sources (in particular, it may be information type X and type Y diametrically opposed to each other). The model describes the dynamics of its evolution and determines its eventual result: which of the sources is a “winner” and which is a “loser”. The winner is the one who at the time of the full coverage of the community managed to extend its information among more members of the community than the opponent. This model assumes that having received the information from one source, an individual is blocked for the other, that is “overpersuasion” is impossible. Also accept three additional factors: 1) incomplete coverage of the society by the media (it is supposed here that some individuals do not use mass media and therefore can receive information only through interpersonal communication with spreaders, while others can receive it either from the media or from spreaders); 2) forgetting of information by individuals; 3) two-stage perception and forgetting of information (it is supposed here that an ignorant individual becomes a pre-spreader after receiving the information. Pre-spreaders do not spread information further. After receiving the information again this individual becomes a spreader). The model has a form of

Keywords: mathematical modeling, information warfare, media propaganda, interpersonal communication, differential equations.

2010 Mathematics Subject Classification: 94A05; 91C99; 65L80
Cauchy problem for a system of ODE:

\[
\frac{dX_1}{dt} = x_1(\alpha x + \beta x(X_1 + X_2)) - \gamma_x X_1, \quad \frac{dX_2}{dt} = \beta x x_2(X_1 + X_2) - \gamma_x X_2 \\
\frac{dY_1}{dt} = y_1(\alpha y + \beta y(Y_1 + Y_2)) - \gamma_y Y_1, \quad \frac{dY_2}{dt} = \beta y y_2(Y_1 + Y_2) - \gamma_y Y_2 \\
\frac{dx_1}{dt} = (\alpha x + \beta x(X_1 + X_2))(N_1 - X_1 - Y_1 - 2x_1 - y_1) + \gamma_x X_1 - \delta_x x_1 \\
\frac{dx_2}{dt} = \beta x(X_1 + X_2)(N_2 - X_2 - Y_2 - 2x_2 - y_2) + \gamma_x X_2 - \delta_x x_2 \\
\frac{dy_1}{dt} = (\alpha y + \beta y(Y_1 + Y_2))(N_1 - X_1 - Y_1 - x_1 - 2y_1) + \gamma_y Y_1 - \delta_y y_1 \\
\frac{dy_2}{dt} = \beta y(Y_1 + Y_2)(N_2 - X_2 - Y_2 - x_2 - 2y_2) + \gamma_y Y_2 - \delta_y y_2 \\
X_1(0) = X_2(0) = Y_1(0) = Y_2(0) = x_1(0) = x_2(0) = y_1(0) = y_2(0) = 0
\]

Here \(x_i, X_i\) - the number of pre-spreaders and spreaders of source \(X\) from group \(i\) (same for source \(Y\)); \(\alpha_x, \beta_x, \gamma_x, \delta_x\) - parameters characterizing accordingly the intensity of information dissemination through the media and through interpersonal communications, forgetting by spreaders and pre-spreaders for source \(X\) (same for source \(Y\)); \(N_i\) - the number of individuals from group \(i\).

This model is studied numerically and analytically. The meaningful interpretation is given for the results obtained.

References


On a stationary solution of a nonlinear singularly perturbed heat equation

M. A. Davydova 1, N. N. Nefedov 2, S.A. Zakharova 3

1, 2, 3 Moscow State University, Moscow, Russia

E-mail: 1m.davydova@physics.msu.ru; 2nefedov@phys.msu.ru, 3sa.zakharova@physics.msu.ru

Abstract  We consider stationary solutions with internal transition layers (contrast structures) in the Dirichlet boundary value problem for the nonlinear singularly perturbed heat equation. We construct an asymptotic approximation of an arbitrary-order accuracy to such solutions and prove the existence theorem. To justify the constructed asymptotics, we use an asymptotic method of differential inequalities, which also permits one to prove the Lyapunov stability of such stationary solutions.

Acknowledgments

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References


Keywords: Reaction-diffusion-advection problems; contrast structure; singularly perturbed problems; asymptotic analyses.

2010 Mathematics Subject Classification: 35K05; 65L11; 35K57.
Neural network model for forecasting of hazard air pollution by PM emitted from peat fire and its effect on road traffic

V. Lozhkin\textsuperscript{1}, V. Timofeev\textsuperscript{1}, O. Lozhkina\textsuperscript{1}, A. Vasilyev\textsuperscript{2}, D. Tarkhov\textsuperscript{2}

\textsuperscript{1} St. Petersburg University of State Fire Service of EMERCOM of Russia, Russia
\textsuperscript{2} Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: vnlojk1n@yandex.ru

Abstract  The present paper describes an original neural network model for the forecasting of hazard air pollution by PM emitted from a peat fire and its effect on the road traffic. The model was developed for a real road transport emergency occurred on the Federal Highway M-255 "Siberia". The model is able to self-learning using experimental and simulated data.

Introduction

The development and the implementation of physical and chemical transportation models is one of the most rapidly developing sections of the modern computational meteorology and atmospheric chemistry and physics [1].

Main results

There was developed a neural network technique for the modeling of pollutants dispersion in the air on meso- and regional scale [2]. The originality of the design of the computational process is determined by the specificity of the object of the investigation - the influence of a peat fire on a busy highway at extreme meteorological conditions. The model predicts transportation of suspended particles PM2.5 and PM10 of the smoke from a peat fire in the vicinity of a highway and involves the estimation of health and traffic risks. The developed functional parametric models allowed us to carry out numerical investigations of emergency situations under different scenarios with different sets of input data [2], in particular, wind speed and direction, heterogeneous experimental and simulated information on the emission intensity of pollutants from peat fire and the concentrations of PM2.5 and PM10. The model was trained using the data of a real transport collapse that occurred in the Irkutsk Region (Russian Federation) in winter 2015-2016 during the burning of the peat near the Federal Highway "Siberia". The transport of particles in the stratified atmosphere is modeled using the differential equation of turbulent diffusion including the sedimentation of PM. The approximate solution of the constructed parametric models was sought in the form of a heterogeneous neural network function, which parameters were found by minimizing the error of functional.

References


Keywords: Neural network model; differential equation; peat fire; particulate matter (PM); road transport; emergency.

2010 Mathematics Subject Classification: 92B20; 90B20.
Cordial Volterra integral equations and singular fractional integro-differential equations in spaces of analytic functions

Urve Kangro

University of Tartu, Estonia

E-mail: urve.kangro@ut.ee

Abstract We study general cordial Volterra integral equations of the second kind and certain singular fractional integro-differential equation in spaces of analytic functions. We characterize properties of the cordial Volterra integral operator in these spaces, including compactness and describe its spectrum. This enables us to obtain conditions under which these equations have a unique analytic solution. We also consider approximate solution of these equations and prove exponential convergence of approximate solutions to the exact solution.

References


Keywords:cordial integral equation, singular fractional differential equation, analytic solution, exponential convergence, collocation method.

2010 Mathematics Subject Classification: 34A08, 45D05, 45J05, 47G20, 65R20.
An approach to multiple attribute decision making based on normal Pythagorean fuzzy aggregation operators

Ridvan Sahin

Bayburt University, Turkey

E-mail: mat.ridone@gmail.com

Abstract  Normal Pythagorean fuzzy numbers (NPFNs) are an important tool to describe the decision making problems, and they are more appropriate to depict complex and uncertain decision making information. In this paper, we propose the definition, the properties, the score function and accuracy function of the NPFNs, and prove some of their operational laws. Then we propose some new aggregation operators, including normal Pythagorean fuzzy weighted averaging (NPFWA) operator, normal Pythagorean fuzzy weighted geometric (NPFWG) operator, normal Pythagorean fuzzy ordered weighted averaging (NPFOWA) operator, normal Pythagorean fuzzy ordered weighted geometric (NPFOWG) operator, normal Pythagorean fuzzy generalized ordered weighted averaging (NPFGOWA) operator and normal Pythagorean fuzzy generalized ordered weighted geometric (NPFGOWG) operator. Moreover, a new multiple attribute decision making method with Pythagorean fuzzy information based on the new aggregation operators is proposed and a practical numerical example is presented to illustrate the feasibility and practical advantages of the new method.

References


Keywords: normal fuzzy set; normal Pythagorean fuzzy set; aggregation operators; decision making.
2010 Mathematics Subject Classification: 03E72, 94D05, 90B50.
Generalizing the MDCA Method for the Search of a Global Optimum of a Nonconvex Function in an $\mathbb{R}^n$ Box

Fadila Leslous$^1$, Philippe Marthon$^2$, Mohand Ouanes$^3$, Mohammed Chebbah$^4$

$^1$Laboratory LAROMAD, Mouloud Mammeri University, Algeria
$^2$IRIT-ENSEEIHT, University of Toulouse, France

E-mail: $^1$fadila-leslous@ummto.dz; $^2$pilippe.marthon@enseeiht.fr; $^3$ouanes-mohand@yahoo.fr; $^4$chebbah-med@ummto.dz

Abstract This paper presents a generalization of the MDCA method for nonconvex functions in an $\mathbb{R}^n$ box, using the DCA (Difference of Convex Algorithm) and the minimum of the average of approximations of the function from the box endings. This strategy has the advantage of giving in general a minimum to be situated in the attraction zone of the global minimum searched. After applying the DCA from this minimum we certainly arrive at the global minimum searched.

References


Keywords: Optimization DC and DCA; Global optimization; Nonconvex optimization.

2010 Mathematics Subject Classification: 90C26; 90C34; 90C25.
Computational investigation of inclusion to some kinds of trihydroxyflavanone with β-cyclodextrin

Safia Himri, Leila Nouar, Fatiha Madi

Laboratory of computational chemistry and nanostructures, Guelma University, Algeria.

E-mail: himri.safia@gmail.com

Abstract  The encapsulation of 4',5,7-trihydroxyflavanone into β-cyclodextrin (β-CD) has been studied theoretically using PM3 and density function theory B3LYP/6-31G(d) methods. Complexation energy, dipole moment, HOMO and LUMO energies structural parameters of two proposed complexes are investigated. The results show clearly that the formed complexes are energetically favored. Finally, natural bonding orbital (NBO) analysis was employed to quantify the donor-acceptor interactions between 4',5,7-trihydroxyflavanone and β-CD.

References


Keywords: β-Cyclodextrin; 4',5,7-trihydroxyflavanone; PM3; B3LYP/6-31G(d); NBO.

2010 Mathematics Subject Classification: 91-08.
Global solutions of Frankl system with sequestered coefficients providing by filling of initial data

Tatiana A. Shemyakina

Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: sh_tat@mail.ru

Abstract Nonlocal solvability of the Cauchy problem is proved for a special case of the Frankl system in physical variables. The investigation of the considered problem is based on the method of an additional argument. The proof of the nonlocal solvability relies on original global estimates.

Introduction

We consider the Frankl system:

\[
\begin{align*}
\partial_x u(x, y) - P(x, y, u(x, y), v(x, y))\partial_y v &= 0, \\
\partial_x u(x, y) &\pm Q(x, y, u(x, y), v(x, y))\partial_y v = 0,
\end{align*}
\]

where \( u(x, y), v(x, y) \) are unknown functions; \( P(x, y, u(x, y), v(x, y)) \geq p_0 > 0, Q(x, y, u(x, y), v(x, y)) \geq q_0 > 0; p_0, q_0 \) are const. For the first time the famous physicist F. Frankl has combined the study of the system of equations of mixed type with the study of stationary problems of transonic gas dynamics. The Frankl system is a system of mixed type. The famous scientists studied problems for systems of mixed type equations in cases, when the equation system coefficients \( P, Q \) possess values of the constants or independent variables. Most often the study came to the study of differential equations of the second order. As a rule, the method of research is to convert the nonlinear equations to linear equations. Then a linear system of equations is solved. But in the general case, a return to the original variables is a more difficult task than the original task. In many cases the problem of inverse function determination is so difficult that it is not solved. The existence of inverse variable transformations is taken as a condition. A variety of approaches to the study of mixed systems of differential equations have their advantages and disadvantages. The method we propose does not replace other known methods. It amplifies them and allows to determine more precisely the conditions for the solvability in the original variables.

Main results

We consider the Frankl system. Under certain conditions, it describes a stationary motion of an ideal gas at the supersonic speeds. The coefficients \( P, Q \) take the following form: \( P(v^2), Q(v^2) \) Considering physical properties of process, we will transform coefficient \( Q \). After the alteration a new coefficient \( Q \) is obtained in the following form: \( Q(v^2) = k_0 v^{-2} P^{-1}(v^2) \), where \( k_0 \) is const. In this case, we call of the Frankl system coefficients sequestered. For the Frankl system we define initial conditions: \( u(x,0) = \varphi(x), v(x,0) = \psi(x), x \in (-\infty, \infty); \varphi(x), \psi(x) \in C^2(R^1) \). The problem is considered of the domain: \( \Omega_Y = \{(x, y) : x \in (-\infty, \infty), y \in [0, Y], Y > 0 \} \). In the works \([1–2]\) authors have

Keywords: Frankl system, system of two quasilinear partial differential equations of the first order, mixed system of equations, method of an additional argument, global estimates.

2010 Mathematics Subject Classification: 35L50, 35M11, 35Q35, 76B03

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developed essentially new method of application of a method of an additional argument to the Frankl system. We have proved existence of the local solutions, the smoothness of which is not lower the smoothness of the initial conditions. In systems of the equations the number of superpositions of unknown functions was reduced. This simplifies the study of tasks. In the work [3] we found sufficient conditions for a nonlocal solvability of the Cauchy problem for a system of two quasilinear partial differential equations of the first order. At that, the system of equations is represented in characteristic form. The proof of the nonlocal solvability of the problem in the original variables is based on the method of an additional argument. We find an original global estimates that guarantees existence of a global classical solution continued from a local solution. However, for the Frankl system with arbitrarily given initial data, conditions for the existence of the global solution established in work [3], are not fulfilled. In the work [4] we prove the global existence of a self-similar solutions of the Frankl system under the condition that the initial data are consistent. The Frankl system has coefficients of the form \( P(u, v), Q(u, v) \). In this paper we have found a global classical solution of the Frankl system with sequestered coefficients and subject to the coordination of the initial data. The study of the problem is performed on the basis of the method of an additional argument. The results are formulated as a theorem.

**Theorem 46.** Suppose \( P(x, y, u(x, y), v(x, y)) \geq p_0 > 0, \ Q(x, y, u(x, y), v(x, y)) \geq q_0 > 0 \) are twice continuously differentiable in all its arguments; \( \varphi(x), \psi(x) \in \mathcal{C}^2([R^1]) \) are the functions subject to the coordination of the initial data \( \varphi(x) = k_0^{-0.5} \ln(\psi(x)) + c; (P(v) + v\partial_x P(v))\psi'(\cdot) \leq 0, c \) is const. Then the Frankl system with sequestered coefficients has a unique bounded on the set \( x \in [R^1] \) solution \( u(x, y) = \varphi(x + yk_0^{-0.5}vP(v)), v(x, y) = \psi(x + yk_0^{-0.5}vP(v)) \).

**Remark** Suppose \( (P(v) + v\partial_x P(v))\psi'(\cdot) > 0 \), then function \( v(x, y) \) goes to infinity on a finite interval.

**References**


The contrast structure type solution of the reaction-diffusion equation in case of discontinuous reactive and diffusive terms

Natalia Levashova\(^1\), Olga Nikolaeva\(^1\), Nikolay Nefedov\(^1\), Andrey Orlov\(^1\)

\(^1\)Faculty of Physics, M.V.Lomonosov Moscow State University, Moscow, Russia

E-mail: \(^1\)natasha@mpanalytica.ru

Abstract A boundary value problem for one dimensional stationary diffusion-reaction equation with small diffusion was concerned on a segment. The reaction and diffusion terms are assumed to undergo discontinuity of the first kind at some internal point of the segment. The existence of continuous contrast structure type solution of the problem was proved. The asymptotic approximation of the solution in powers of small parameter was constructed.

Introduction

We are concerned with the boundary value problem of reaction-diffusion type

\[
\varepsilon^2 \frac{d}{dx} \left( k(x) \frac{du}{dx} \right) = f(u, x, \varepsilon), \quad x \in (-1; 1), \quad \frac{du}{dx}(-1) = u_0^-(1), \quad \frac{du}{dx}(1) = u_0^+(1),
\]

where \( \varepsilon \in (0; \varepsilon_0) \) is a small parameter. We assume that the function \( k(x) \) is positive on a segment \( x \in [-1; 1] \), and function \( f(u, x, \varepsilon) \) is defined when \( u \in I_u, x \in [-1; 1], \varepsilon \in (0; \varepsilon_0) \) where \( I_u \) is allowed interval of variable \( u \) values. We assume that there exists an internal point \( x_0 \), of the segment \([-1; 1]\), at which the function \( k(x) \) may undergo the discontinuity of the first kind:

\[
k(x) = \begin{cases} 
  k^-(x), & -1 \leq x \leq x_0; \\
  k^+(x), & x_0 \leq x \leq 1,
\end{cases}
\]

and the functions \( k^-(x) \) are smooth enough at the segments \([-1; x_0]\) and \([x_0; 1]\), respectively. The function \( f(u, x, \varepsilon) \) undergoes the discontinuity of the first kind along the line \( \{u \in I_u, x = x_0\} \):

\[
f(u, x, \varepsilon) = \begin{cases} 
  f^-(u, x, \varepsilon), & u \in I_u, -1 \leq x \leq x_0; \\
  f^+(u, x, \varepsilon), & u \in I_u, x_0 \leq x \leq 1.
\end{cases}
\]

and functions \( f^-(u, x, \varepsilon) \) are smooth enough on the set \( I_u \times [-1; x_0] \) and \( I_u \times [x_0; 1] \) respectively.

Required conditions

\textbf{C1.} Let the equation \( f^-(u, x, 0) = 0 \) has an isolated solution \( u = \varphi^-(x) \) at the segment \([-1; x_0]\) and the equation \( f^+(u, x, 0) = 0 \) has an isolated solution \( u = \varphi^+(x) \), at the segment \([-1; x_0]\) and the following inequality holds: \( \varphi^+(x_0) < \varphi^-(x_0) \).

\textbf{C2.} Let the inequality \( f_u(\varphi^-(x, 0), x) > 0 \) holds for \(-1 \leq x \leq x_0\) and the inequality \( f_u(\varphi^+(x, 0), x) > 0 \) holds for \( x_0 \leq x \leq 1 \).

Keywords: Reaction-diffusion equation; boundary value problem; asymptotic approximation.

2010 Mathematics Subject Classification: 35K57; 30E25; 41A60.
We consider the solution of a contrast structure type. That is a function close to \( \phi^{-}(x) \) inside the vicinity of the point \( x_0 \) when \( x < x_0 \) and \( \phi^{+}(x) \) inside the vicinity of the point \( x_0 \) when \( x > x_0 \) and undergoing a rapid change in the vicinity of point \( x_0 \). Also we require that the solution satisfies the following conjugation conditions:

\[
k^{-}(x_0) \frac{du}{dx}(x_0 - 0) = k^{+}(x_0) \frac{du}{dx}(x_0 + 0).
\] (178)

**C3.** Let the inequalities

\[
\int_{\phi^{-}(x_0)}^{p} f^{-}(u, x_0, 0) du > 0, \quad \int_{\phi^{+}(x_0)}^{p} f^{+}(u, x_0, 0) du > 0
\]

hold for \( \phi^{-}(x_0) < p < \phi^{+}(x_0) \).

**C3.** Let there exist a value \( p_0 \in (\phi^{-}(x_0), (\phi^{+}(x_0)) \) that is the solution of the equation

\[
k^{-}(x_0) \int_{\phi^{-}(x_0)}^{p_0} f^{-}(u, x_0, 0) du - k^{+}(x_0) \int_{\phi^{+}(x_0)}^{p_0} f^{+}(u, x_0, 0) du = 0.
\]

**C4.** Let the following condition hold

\[
k^{-}(x_0) f^{-}(p_0, x_0, 0) - k^{+}(x_0) f^{+}(p_0, x_0, 0) > 0.
\]

**Main result**

Under the formulated conditions the existence of the solution of problem (177) was proved. The asymptotic approximation of the solution in powers of small parameter \( \epsilon \) was constructed.

**Acknowledgments**

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**References**


An efficient computational method based on Legendre wavelet for partial integro-differential equations arising from viscoelasticity

Vineet Kumar Singh

Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University) Varansi, India

E-mail: vksingh.mat@iitbhu.ac.in

Abstract In this study, we developed a computational method based on Legendre wavelet and their operational matrices for the approximate solution of partial integro-differential equation (PIDE). This problem can be found in the mathematical modeling of physical phenomena involving viscoelastic forces. By implementing the two dimensional Legendre wavelets and the associated operational matrix of integration the given PIDE reduces to the system of algebraic equations which can be solved by some well known methods. Convergence analysis, numerical stability and rate of convergence (C-order) of the proposed method are also investigated by considering a test function. Numerical results confirm the predicted convergence rates and also exhibit optimal accuracy in the $L^2$ and $L^\infty$ norms.

References


Keywords: Partial integro-differential equation; Legendre wavelet; operational matrix of integration; Convergence analysis.

2010 Mathematics Subject Classification: 45J05; 60J60; 42C40.
Higher degree fuzzy transforms based on B-splines

Martins Kokainis ¹, Svetlana Asmuss ²

¹University of Latvia, Latvia ²Institute of Mathematics and Computer Science, Latvia

E-mail: ¹martins.kokainis@lu.lv; ²svetlana asmuss@lu.lv

Abstract

We consider the continuous and discrete higher degree fuzzy transforms (F-transforms with polynomial components) with respect to a generalized fuzzy partition given by B-splines. The obtained results on approximation properties of the direct and inverse F-transforms in these cases show that using B-splines allows to improve the quality of approximation of smooth functions and their derivatives. We investigate conditions under which the higher degree fuzzy transforms preserve such special properties of functions as monotonicity and convexity. The results are generalized to the multidimensional case by considering approximation of multivariate functions.

The concept of fuzzy transform (F-transform or \(F^0\)-transform) was introduced in 2001 [1] (see also the key paper [2]) and generalized to the case of higher degree (F-transform with \(m\)-degree polynomial components or \(F^m\)-transform) in 2011 [3] by I. Perfilieva with co-authors. The core of the fuzzy transform technique consists of partitioning the universe using fuzzy sets. A quality of the technique of F-transforms depends on basic functions used for fuzzy partitioning. Our research focuses on investigation of properties of the direct and inverse \(F^m\)-transforms \((m \geq 0)\) with respect to a generalized uniform fuzzy partition given by B-splines of an odd degree.

Suppose that \([a, b] \subset \mathbb{R}\) and \(N, k \in \mathbb{N}\) are chosen. Let \(t_i = a + hi\) for \(i \in \{0, \ldots, N\}\), where \(h = (b - a) / N\). For \([a, b]\) we consider the fuzzy partition \(A_0, \ldots, A_N : [a, b] \rightarrow [0, 1]\) given by basic functions \(A_i(t) = A((t - t_i)/h)\), where \(A\) is the central B-spline of degree \(2k - 1\) with the integer knots \(-k, \ldots, 0, \ldots, k\). We note that such fuzzy partition fulfills the Ruspin condition (the sum of all basic functions is equal to 1) on the interval \([t_{k-1}, t_{N-k+1}]\).

For each \(i \in \{0, \ldots, N\}\) denote by \(L_2(A_i)\) the Hilbert space of all square integrable functions \(f : supp(A_i) \rightarrow \mathbb{R}\) with the inner product \(\langle f, g \rangle_i = \int_{\mathbb{R}} f(t)g(t)A_i(t)dt\).

Let \(f : [a, b] \rightarrow \mathbb{R}\) be a function s.t. \(f|_{supp(A_i)} \in L_2(A_i)\) for all \(i \in \{0, \ldots, N\}\). The orthogonal projection of \(f|_{supp(A_i)}\) on \(L_2^m(A_i)\) is denoted by \(F_m^-|_{f}\) (here \(L_2^m(A_i)\) is the closed subspace of \(L_2(A_i)\), spanned by the set of polynomials \(\{1, t, t^2, \ldots, t_m\}\)). Then the vector \(F_m[f] = (F_m^-|_{f}, \ldots, F_m^-|_{f})\) is said to be the direct \(F^m\)-transform of \(f\) w.r.t. the fuzzy partition \(A_0, \ldots, A_N\).

If \(p = (p_0, \ldots, p_N)\) be a tuple of \(N + 1\) polynomials of degree at most \(m\), then the function

\[F_m^-|p|(t) = \frac{\sum_{i=0}^{N} p_i(t) A_i(t)}{\sum_{i=0}^{N} A_i(t)}\]

is called the inverse \(F^m\)-transform of \(p\) w.r.t. the fuzzy partition \(A_0, \ldots, A_N\). Now we apply to \(f\) the composition of the direct and inverse \(F^m\)-transforms and obtain function \(F_m[f]\) called the \(F^m\)-transform of \(f\) w.r.t. the fuzzy partition \(A_0, \ldots, A_N\):

\[F_m[f](t) = \frac{\sum_{i=0}^{N} F_m^-|_{f}(t) A_i(t)}{\sum_{i=0}^{N} A_i(t)}\]

Keywords: fuzzy partition; B-spline; fuzzy transform; approximation error; monotone approximation; convex approximation.

2010 Mathematics Subject Classification: 03E72; 41A30; 65D07; 68T30.
We have proved that the $F^m$-transform based on B-splines of degree $2k - 1$ is precise for polynomials of degree $r \leq \min\{2m+1, 2k-1\}$ on the interval $[t_{N-k+1}, t_{N}]$. On the basis of this result we obtain the following error estimation

$$\| f^{(n)}(t) - (F^m f)^{(n)}(t) \| = O(h^{r-n+1/q'})$$

for functions $f$ from $L^{r+1}_q([a,b])$ (i.e., s.t. $f^{(r+1)} \in L^q([a,b])$, when $q \geq 1, 1/q+1/q' = 1, 0 \leq n \leq r \leq \min\{2m+1, 2k-1\}$ and $t \in [t_{N-k+1}, t_{N}]$). The result $O(h^{2m+2})$ is achieved using B-splines of degree $2m + 1$ (or greater), when we consider the approximation error for functions from $C^{2m+2}([a,b])$. This improves the best estimation $O(h^{m+1})$ on the approximation error, which is known for the $F^m$-transform with respect to an arbitrary uniform fuzzy partition with the parameter $h$.

We consider generalizations of this result for the discrete and multidimensional versions of fuzzy transforms (one can find particular results in [4] and [5]). We generalize also some results from [6] and obtain conditions under which the higher degree fuzzy transforms based on B-splines preserve such special properties of functions as monotonicity and convexity.

References


Approximate numerical solution of fractional delay differential equations using operational matrix method

Harendra Singh

School of Mathematical Sciences, National Institute of Science Education and Research (NISER), Khurda-752050, Odisha, India

E-mail: harendrasingh@niser.ac.in

Abstract In this paper, we have presented an approximate method for solving fractional delay differential equations (FDDEs). The approximate method is based on the application of the operational matrix of integration of the Chebyshev polynomial of fourth kind. The operational matrix approach converts the FDDE into a set of linear system of equations, and solving the linear system, the approximate solution of the FDDE is obtained. Maximum absolute error (MAE) and root mean square error (RMSE) are calculated for the illustrated example and presented in form of tables for the comparison purpose. Numerical stability of the presented method is provided. The obtained numerical results are also compared with some known methods from the literature.

Keywords: Fractional delay differential equations; Chebyshev polynomials; operational matrix; numerical stability.

2010 Mathematics Subject Classification: 45J05; 60J60; 42C40.

References


An analogue of the Schwarz problem for the Moisil–Teodorescu system in multiply connected domains

V.A. Polunin, A.P. Soldatov

Belgorod State University, Belgorod, Russian Federation

E-mail: polunin@bsu.edu.ru; soldatov@bsu.edu.ru

Abstract An analogue of the Schwarz problem for the Moisil–Teodorescu system is considered in a domain $D$. It is shown that this problem is Fredholm in the Holder class $C^\mu(D)$. If the domain $D$ is homeomorphic to a ball then the problem is investigated in detail. In particular its index is equal to $-1$ in this case.

References


Keywords: Moisil–Teodorescu system; Schwarz problem; Cauchy type integral; singular integral equation.

2010 Mathematics Subject Classification: 35J25, 35J55; 45F15.
Magnetogasdynamic shock wave propagation in self-gravitating gas

G. Nath¹, Sumeeta Singh²

¹MNNIT Allahabad, India ²MNNIT Allahabad, India

E-mail: ¹gn_chaurasia_univgkp@yahoo.in; ²sumeeta2304bhadauria@gmail.com

Abstract Mathematical modeling for the propagation of plane or cylindrical shock wave in a self-gravitating ideal gas with transversed magnetic field is done. Distribution of hydrodynamical quantities are discussed. The density is assumed to be varying according to power law with distance. Following Sakurai [1] and Siddiqui, Arora and Kumar [2], solutions are obtained in the form of power series in \((C^2)^2\), where \(C\) is the sound velocity of undisturbed fluid. First approximations are considered for both the cases of plane and cylindrical shock waves. The effect of magnetic field and self-gravitation of gas are studied on shock strength and flow variables.

Fundamental Equations

The fundamental equations governing the one-dimensional planar or cylindrically symmetric flow of a self-gravitating ideal gas in the presence of an transverse magnetic field are (c.f. [1][2])

\[
\rho_t + \rho u_r + u \rho_r + \frac{i \rho u}{r} = 0, \tag{179}
\]

\[
u_t + uu_r + \gamma p(p_r + h_r) + \frac{Gm}{r^i} = 0, \tag{180}
\]

\[
p_t + up_r + \gamma p \left( u_r + \frac{iu}{r} \right) = 0, \tag{181}
\]

\[
h_t + uh_r + 2h \left( u_r + \frac{iu}{r} \right) = 0, \tag{182}
\]

where \(\rho\) is density, \(u\) is particle velocity, \(p\) is pressure, \(h\) is magnetic pressure given by \(h = \frac{\mu S^2}{2}\) with \(\mu\) as magnetic permeability and \(S\) is the transverse magnetic field, \(\gamma\) is specific heats ratio of the gas, \(m\) is mass, \(G\) is gravitational constant; \(t\) and \(r\) are independent time and space coordinates with \(i = 0, 1\) for the respective cases of plane and cylindrical shock.

Significance of Study

Study of cylindrical shock waves is not only associated with the explosion of a long thin wire but also to certain axially symmetrical hypersonic flow problems, such as the shock envelope behind a fast meteor, or missile. As an example, when a meteor, or a hypersonic missile, is shooting through the atmosphere at great speed, the shock envelope at distances sufficiently far behind the flying object can be considered as locally one-dimensional (see Shao-Chi Lin [3]).

Keywords: Shock wave; magnetic field; self-gravitation; power series solution.

2010 Mathematics Subject Classification: 76L05; 76W05; 35Lxx.
Astrophysical systems—planets, stars, galaxies, galaxy clusters, and possibly the intergalactic medium at large—carry magnetic fields. As the particles of plasma are charged, they are strongly influenced by electromagnetic effects. Gravity is the dominant interaction at the macroscopic scale, and is the cause of formation, shape and trajectory of astronomical bodies.

Acknowledgments

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References


Application of orthogonal polynomials for generalized Abel integral equations via operational matrices

Ashan Gupta\textsuperscript{1}, R. K. Pandey\textsuperscript{2}

Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University) Varansi, India

E-mail: ishangupta903@gmail.com; rajeshpbhu@gmail.com

Abstract This work provides a comparative study of four numerical schemes based on orthogonal polynomials for the approximate solution of generalized Abel integral equations. Operational matrices of integration of orthogonal polynomials combined with collocation method are used to reduce the solution of Abel integral equation into the system of linear equations which can be solved by some well known iterative solvers. Convergence analysis and numerical stability of the proposed schemes are provided under several mild conditions. Numerical experiments suggest the proposed schemes achieve good accuracy and efficiency.

References


Keywords: Abel integral equation; orthogonal polynomials; operational matrix of integration; convergence analysis.

2010 Mathematics Subject Classification: 31A10; 33C50.
Classification of mammographic masses according to their shapes in digitized mammograms

Chokri Ferkous¹, Brahim Guerzize¹, ²Hayet Farida Merouani

¹LabSTIC, Guelma University, Algeria; ²LRI, Badji Mokhtar University Annaba, Algeria

E-mail: ferkous.chokri@yahoo.fr

Abstract In this paper, we propose a computer-aided diagnosis system to extract features and classify mammographic masses to differentiate between round, lobulated and irregular shapes in digitized mammograms; this system is inspired overall by the approach of the doctor during the radiologic examination as it was agreed in BI-RADS (Breast Imaging Reporting System and Data System), where mammographic masses are described by their shape, their edge and their density. The segmentation of masses in our approach is manual because it is supposed that the detection is already made. When the segmented region is available, features extraction process can be carried out. Thereafter it is sought to detect the contours of the mass using a genetic active contour, the best snake of the population at a given instant is considered as approximating contour of the abnormality. Energies of continuity, curvature and irregularity of the final snake, in addition to the size of the abnormality form the features vector. Classification is finally done using a Multi-Layer Perceptron (MLP). The proposed approach has been evaluated on 150 mammographic masses extracted from the Digital Database for Screening Mammography (DDSM), and the obtained results are encouraging.

References


Keywords: Active contours; genetic algorithms; genetic active contour; medical image; mammography; ACR; BI-RADS.

2010 Mathematics Subject Classification: 92D15; 68R01; 68T10; 68T45.
Derivative-free variants of King's family with memory having high efficiency index for solving nonlinear equations

Munish Kansal*, V. Kanwar

University Institute of Engineering and Technology, Panjab University, Chandigarh, India

E-mail: *mkmaths@gmail.com

Abstract In this paper, we introduce a new efficient class of higher-order Steffensen-type methods with memory for solving nonlinear equations numerically. Firstly, we construct a two-point class of derivative-free methods without memory based on King’s family using weight function approach. The proposed family requires only three functional evaluations to achieve optimal fourth order of convergence. Furthermore, acceleration of convergence speed is attained by suitable variation of free parameters in each iterative step. These self-accelerating parameters are estimated from current and previous iterations using Newton’s interpolation polynomials of third and fourth-degrees. Consequently, the R-order of convergence is increased from 4 to 7. As a result, efficiency index raises from $E = \sqrt[4]{4} \approx 1.587$ to $E = \sqrt[7]{4} \approx 1.913$, which is even better than optimal sixteenth order methods without memory. The most interesting feature of the presented methods with memory is that convergence order is increased without adding any extra functional evaluation. Therefore, the proposed methods with memory possess a very high computational efficiency. Finally, numerical experiments on some practical problems like Planck’s radiation law problem which calculates the energy density within an isothermal blackbody are included to confirm the theoretical results and high efficiency index in multi-precision computing environment.

References

On numerical solving time-fractional differential equations

Mesut Karabacak¹, Ebubekir Karabacak²

¹Atatürk University, Turkey  ²Atatürk University, Turkey

E-mail: ¹mkarabacak@atauni.edu.tr; ²ebubekir.karabacak@atauni.edu.tr

Abstract   In this manuscript, the numerical solution of some known time-fractional differential equations is considered by some known effective numerical methods with different types of fractional derivative definitions comparatively. Illustrative examples are presented to demonstrate the applicability and validity of those numerical methods under different fractional derivative definitions with tables and figures of the numerical solutions.

References


Investigation of natural convection of a non-Newtonian nanofluid flow between two vertical flat plates using Birkhoff interpolation method

Ghasem Barid Loghmani\textsuperscript{1}, Nasibeh Karamollahi\textsuperscript{2}, Mohammad Heydari\textsuperscript{3}

\textsuperscript{1,2,3} Yazd University, Iran

E-mail: \textsuperscript{1}loghmani@yazd.ac.ir; \textsuperscript{2}Karamollahi.n@gmail.com; \textsuperscript{3}m.heydari@yazd.ac.ir

Abstract In this work, a numerical scheme based on Birkhoff interpolation method (BIM) is proposed and used to investigate the problem of natural convection of a non-Newtonian nanofluid flow between two vertical flat plates. The effect of some physical parameters on velocity and temperature profiles is considered. Furthermore, the numerical results of the proposed method are compared with those obtained by fourth-order Runge-Kutta method, least square method and differential transformation method in order to confirm the efficiency and accuracy of the method presented in this paper. This method can be easily applied for solving other linear and non-linear equations on the interval $[-1,1]$ and it can be also extended to an arbitrary interval $[a,b]$. Therefore, BIM can be extensively applicable in engineering and science.

References


Optimization with Defaultable Securities

Yaacov Kopeliovich

1 University of Connecticut Department of Finance Storrs 06269

E-mail: yaacov.kopeliovich@uconn.edu

Abstract  I am going to explain how to formulate portfolio optimization problem in presence of bankruptcy and produce an analytical recursive solution for portfolio of bonds with fixed coupon and random bankruptcy.

References


Keywords : Merton Dynamic Optimization Problem, Value Functions, Poisson Distribution
2010 Mathematics Subject Classification : 97M30;62P05.
Identifiability Algorithm for the Differential Equation with Memory and Loaded Masses on Graphs

Sergei Avdonin\textsuperscript{1}, Karlygash Nurtazina\textsuperscript{2}

\textsuperscript{1}University of Alaska Fairbanks, USA \hspace{1em} \textsuperscript{2}L.Gumilyov Eurasian National University, Kazakhstan

E-mail: \textsuperscript{1}s.avdonin@alaska.edu; \textsuperscript{2}nurtazina.k@gmail.com

Abstract  The differential equation with memory with a source-like term is defined on each edge on the graph, and point masses attached to the interior vertices. The response operator uniquely determines the sources, lengths of the edges, attached masses, and the topology of the graph. The proofs are based on the boundary control method and the leaf peeling method for the differential equation with memory.

References


Keywords: differential equation with memory; source identification; graph-tree; boundary control method; leaf peeling method.

2010 Mathematics Subject Classification: 35R30; 35R02; 35P30.
Fractional differential model of percolative phonon-assisted hopping in mesoporous semiconductor

Renat T. Sibatov, Vadim V. Shulezhko, Vyacheslav V. Svetukhin

Ulyanovsk State University, Russia

E-mail: ren_sib@bk.ru

Abstract In this report, we discuss and verify the fractional differential model of phonon-assisted hopping in mesoporous semiconductor. The model is formulated in terms of the continuous time random walk on percolation cluster of the Skal-Shklovskii-De Gennes type and accounts for combined effects of localized states with distributed energies and percolative character of carrier trajectories. To verify predictions of the model, we perform a series of Monte Carlo simulations of phonon assisted hopping in mesoporous TiO$_2$ films. To elucidate the role of directed percolation in combined effect of structural and energetic disorder, transient current curves of the time-of-flight method are calculated and analyzed.

Introduction

The combined account for energy disorder and percolation is actual for description of charge transport in dye-sensitized solar cells (DSSC), which traditionally include a highly porous metal oxide semiconductors, such as TiO$_2$ film on a transparent conductive electrode. The efficient electronic transport is responsible for overall performance of DSSC and disordered structure of porous anode affects essentially on transport characteristics. Electron transport in mesoporous TiO$_2$ exhibits main features of dispersive transport in disordered semiconductors [1]. The models of phonon-assisted hopping (PAH) and multiple trapping (MT) are often used for description of charge transport in DSSC. In frameworks of both models, fractional diffusion equations of dispersive transport were obtained previously [2]. In this paper, we examine modification of these fractional models accounting for combined effects of energetic and spatial disorder.

Main results

Considering Continuous Time Random Walk on the percolation Skal-Shklovskii-De Gennes cluster, we arrive at the following Laplace transform of the generalized Fokker-Planck equation

\[
{c_β}(s^β Ψ(s) + γ)^β - c_β γ^β \hat{ρ}(x, s) - \hat{L}_{FP} \hat{ρ}(x, s) = G(x) \hat{Ψ}(s).
\]

Here, $\hat{L}_{FP}p(x, t) = \frac{∂}{∂x} \left\{ μEp(x, t) - D \frac{∂}{∂x} p(x, t) \right\}$ is the Fokker-Planck operator, $E$ is the electric field, $μ$ and $D$ are the effective mobility and diffusion coefficient, respectively. Here, we consider constant and homogeneous $μ$ and $D$. The localization time distribution $Ψ(t)$ is related to the density of states $Ψ(t) = ∫^∞_0 \exp \left\{ -ω_0 t e^{-ε/kT} \right\} ρ(ε) dε$. Parameters $β$ and $γ$ are related to tempered power law distribution of visited sites in ‘dead ends’ of a percolation cluster.

Keywords: anomalous diffusion, hopping, fractional equation, dispersive transport, mesoporous semiconductor, polymer blend, percolation

2010 Mathematics Subject Classification: 82C43, 82C80, 60K40, 82C41.
We invert equation (183) for particular situations and consider three important cases: the FP equation containing fractional Riemann-Liouville derivative, tempered fractional derivative, and distributed-order derivative. Solving these generalized FP equations for the time-of-light method conditions, we calculate transient current curves $I(t)$. To verify predictions of the model, we perform a series of Monte Carlo simulations of phonon assisted hopping in mesoporous TiO$_2$ films. Two- and three-dimensional patterns of nanoparticles are generated as a set of $N_n$ spheres with centers randomly distributed over a sample. Each sphere has a random radius from Gaussian distribution with the mean value $\bar{R}$ and standard deviation $\delta_R$. Circles can overlap each other. Porosity of an obtained agglomerate is calculated as a fraction of an empty area. We consider samples of size $5 \times 5 \ \mu m^2$, and particles with the mean radius $\bar{R} = 120 \ \text{nm}$. Hopping is simulated using the Miller-Abrahams relation for the transition rate between localized states with Gaussian or exponential DoS and distributed in nanoparticles. Electron (or hole) makes a hop with the Miller-Abrahams rates $\gamma_{jk} = \Gamma_{jk} \exp\left(-2 \frac{r_{jk}}{a}\right)$, $\Gamma_{jk} = \Gamma_0 \exp(-\Delta U_{jk}/k_B T)$, where $r_{jk}$ is a distance between states, $a$ is a localization radius. Amplitude of hopping rates depends on difference between energies of localized states $\Delta U_{jk} = \varepsilon_j - \varepsilon_k + eF r_{jk}$. Transition occurs into the $k$-th state with minimal $\tau_{j\rightarrow k}$. Under conditions of the ToF-method, carriers are initially generated near the left electrode.

MC simulation of charge carrier hopping in mesoporous samples have shown that topological traps plays a crucial role in dispersion of charge propagation for all values of electric field. These traps act independently of thermalization process. Due to this structural disorder dispersive transport can be observed even at small values of energy disorder, when the DoS width is smaller than $kT$. Theoretical estimations based on CTRW on a percolation cluster characterized by ‘dead-ends’ confirms these results. Topological traps can be responsible for separation of particles into two groups of ‘fast’ and ‘slow’ carriers. This separation is deepened at higher fields.

For nanoparticle agglomerate we have obtained universal transient current curves after averaging over disorder realizations. A universal behavior of the transient photocurrent indicates the self-similarity of charge carrier propagation. The kinetic equation containing derivative of a fractional order has been derived in [2] from the self-similarity property. The fractional differential model allows to describe normal and dispersive transport within a unified formalism. For the case of exponential DoS, the observed dispersion parameter is equal to product of energy dispersion parameter $\alpha = kT/\varepsilon_0$ and structural dispersion parameter $\beta$. For high levels of porosity, parameter $\beta$ is very sensitive to electric field intensity.

Acknowledgments

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References


Invariant surfaces and first integrals of polynomial systems

Valery Romanovski

University of Maribor and CAMTP, Slovenia

E-mail: Valerij.Romanovskij@um.si

Abstract We give an introduction to algorithms of the elimination theory and methods for solving of polynomial algebraic systems and show how they can be used for the qualitative investigation of autonomous systems of ordinary differential equations. An application to the study of the May-Leonard system, which models some ecological and chemical processes, is considered in more details. Some first integrals and periodic solutions in the system are found. Hopf bifurcations are discussed.

Introduction

In this work we discuss the problem of finding invariant surfaces and first integrals in polynomial systems of ODEs depending on many parameters using as the example the May-Leonard asymmetric system

$$\begin{align*}
\dot{x} &= x(1 - x - \alpha_1 y - \beta_1 z), \\
\dot{y} &= y(1 - \beta_2 x - y - \alpha_2 z), \\
\dot{z} &= z(1 - \alpha_3 x - \beta_3 y - z)
\end{align*}$$

where $\alpha_i$, $\beta_i$ ($1 \leq i \leq 3$) are non-negative parameters.

Main results

A well-known and widely used approach for the search of oscillations in models describes by ODEs is to find singular points of the system and then to determine periodic solutions arising as the result of the Hopf bifurcation. However an intrinsic feature of differential systems depending on many parameters is that already the determination of their singular points can be extremely difficult and often unsolvable problem. It is also difficult or impossible to determine systems with first integrals in a given family of systems depending on parameters.

We propose an algorithmic way to find subfamilies admitting a polynomial partial integrals, that is, having an invariant algebraic surfaces. Knowing such surface in some cases we can find families of periodic solutions arising not due to a Hopf bifurcation, but provided by the so-called Lyapunov theorem on holomorphic integral.

This work was supported by the Slovenian Research Agency (research core funding No. P1-0306).

Keywords: polynomial systems of ODEs; invariant manifolds; periodic solutions.

2010 Mathematics Subject Classification: 34C25; 34C60; 37N25.
References


Functional-type a Posteriori Error Estimates and Adaptive Algorithms for Solid Mechanics in 2D

Maxim Frolov

Peter the Great St. Petersburg Polytechnic University, Russia

E-mail: frolov.me@spbstu.ru

Abstract This work is devoted to recent achievements in a posteriori error control by the functional approach that is proposed by Prof. S. Repin and his colleagues. The approach yields reliable error bounds that are valid for all conforming solutions of problems. It requires construction of a set of new additional variables. It is shown that a natural choice of conforming finite element approximations in the Hilbert space $H(div)$ for the additional variables provides efficient implementations of the error control.

Introduction

The question of construction of reliable approaches to error control is one of the key questions in modern numerical analysis. Although, the amount of literature on a posteriori error control and adaptive algorithms is growing rapidly, this research is far from completion and clear conclusions. Functional approach we deal with is robust and promising, particularly for control of approximations obtained via commercial software for engineering. It yields reliable error bounds that are valid for all conforming solutions of problems without any additional assumption about methods used for numerical implementations (see, for instance, [1], [2], [3] for a review).

This report is restricted to problems of solid mechanics in 2D. Respective error majorants have the following form:

$$|||e||| \leq M := D(\tilde{u}, s^*) + R(s^*) + \text{penalty terms}, \quad e := u - \tilde{u},$$ (184)

where $u$ is the exact solution, $\tilde{u}$ is some approximation, $e$ is the corresponding error, $s^*$ is a set of auxiliary variables, and $|||...|||$ denotes the energy norm of the error. Term $D$ represents errors in constitutive relations, $R$ is a residual term with mesh-independent constants. The estimate (184) may contain optional penalty terms that violate the symmetry condition in a weak form. The right-hand side of (184) depends only on the known data and it can be calculated explicitly. This estimate is exact in the sense that the equality is possible to be achieved with a proper setting of parameters and variables. For classical linear elasticity and Cosserat elasticity, all auxiliary fields can be constructed on a basis of finite elements suitable for space $H(div)$ – the Hilbert space of square summable vector-functions with square summable divergence.

Main results

Modern adaptive algorithms for finite element methods consist of four main steps: solve, estimate, mark and refine. Concerning [154] the procedure can be specified as follows:

1. **solve** means compute $\tilde{u}$ on a current finite element mesh;
2. **estimate** means compute (184) from individual loads to elements;

Keywords: finite element methods; a posteriori error estimates; computational mechanics; adaptive algorithms.
2010 Mathematics Subject Classification: 65N15; 65N30; 65N50.
3. **mark** means *mark elements of a mesh with large local errors by some marking strategy*; 
4. **refine** means *divide marked elements and locally refine a mesh*. 

A physically correct choice of free variables in functional-type error estimates and selection of modern finite elements allow obtaining accurate guaranteed upper error estimates. The functional approach does not impose significant additional restrictions on free variables. Exact satisfaction of equilibrium equations is unnecessary. Efficiency of the above technique is shown by numerical examples including consequent mesh adaptations as in [4] and [5].

**Acknowledgments**

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**References**


POSTERS
Fractional Spiking Neurons: a Review

Carla M.A. Pinto\textsuperscript{1}, António Sobral\textsuperscript{2}

\textsuperscript{1}School of Engineering, Polytechnic of Porto \hspace{1cm} \textsuperscript{2}Porto, Portugal

E-mail: \textsuperscript{1}cap@isep.ipp.pt

Abstract  We review simulations of a system of two fractional-order spiking neurons. The internal dynamics of the neurons is modelled by the space-clamped Hodgkin-Huxley equations. The coupling is diffusive and is done in the voltage term. Moreover, the neurons may be symmetrically or asymmetrically coupled. Interesting features are observed in both the symmetric and asymmetric coupled systems. Namely: in-phase synchronization, out-of-phase synchronization, mixed-mode oscillations, small oscillations, localization and relaxation oscillations. The order of the fractional derivative, $\alpha$, introduces 'complexity' in the model, in the sense that broadens the expected behavior observed for the integer-order coupled system. Biologically, $\alpha$ may explain certain differences in processing similar tasks in the human brain.

Introduction

In 1952, Hodgkin and Huxley \cite{hodgkin1952} conducted experiments, in the squid axon, aimed at a better understanding of the mechanisms and rules governing the flow of the electric current in a nerve cell, during an action potential. The derived equations, known as the Hodgkin-Huxley (HH) equations, have had a decisive influence in the understanding of the neuronal function since then.

In-phase solutions are characterized by synchronized firing of the two neurons, i.e., the neurons fire at the same time with the same amplitude. Anti-phase solutions occur when the two neurons fire with a shift of one-half period. Localized solutions partition the systems of oscillators in two distinct sets, one with high amplitudes and the other with small amplitudes. Solutions defined by long periods of quasi-static behavior interspersed with short periods of rapid transition are known as relaxation oscillations. Mixed-mode oscillations are solutions combining traits of relaxation oscillations and small oscillations.

We review the results obtained for a fractional-order model of two coupled HH equations. The equations are coupled diffusively through the voltage term. The coupling may be symmetric, when the two coupling constants are equal, or asymmetric for distinct values of the coupling.

Fractional Calculus - review

Fractional calculus (FC) generalizes differentiation and integration of integer order. Fractional systems have been widely applied to solve real problems in engineering, biology, physics, to name a few \cite{2,3}. The most commonly used fractional-order derivatives are the Caputo, the Grünwald-Letnikov, and the Riemann-Liouville derivatives \cite{2}. The GL derivative is given by the equation \cite{185}

$$
(D^{\alpha}_{GL})f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{k=0}^{[t/a]} (-1)^k \binom{[t/a]}{k} f(t - kh), \quad t > a, \alpha > 0 \tag{185}
$$

\textbf{Keywords}: spiking, neurons, fractional, Hodgkin-Huxley equations, patterns

\textbf{2010 Mathematics Subject Classification}: 34G25; 92B05.
Conclusions

We analyzed a fractional-order system of two coupled Hodgkin-Huxley equations. In the symmetric case, i.e., $k_1 = k_2$, fully synchronized periodic solutions are seen for positive values of the coupling. Moreover, the firing amplitude of the neurons decreases with $\alpha$. I.e., the periodic solution loses stability and a stable equilibrium appears through a Hopf bifurcation, as $\alpha$ is decreased. For negative values of the coupling, anti-phase periodic solutions are ubiquitous, due to symmetry. Nevertheless, when $\alpha$ decreases these solutions are lost by a, probable, symmetry-breaking bifurcation. The asymmetric coupled neurons (i.e., $k_1 \neq k_2$) provide a broader set of patterns. One can distinguish mixed-mode oscillations, small oscillations, relaxation oscillations, and localized solutions, for certain parameters (coupling constants) regions. Moreover, the value of $\alpha$ adds more complexity to the asymmetric model. This may explain differences in the human brain when storing and processing memories, or when reacting to the same stimuli.

Acknowledgments

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References


Multi-quasielliptic and Gevrey regularity of hypoelliptic differential operators

Ahmed Dali\textsuperscript{1}, Chikh Bouzar\textsuperscript{2}

\textsuperscript{1}University of Bechar, Algeria \textsuperscript{2}University of Oran, Algeria

E-mail: \textsuperscript{1}ahmed_dali2005@yahoo.fr; \textsuperscript{2}bouzar@yahoo.com

Abstract In this work, our main aim is to study optimal multi-anisotropic Gevrey regularity of some multi-quasielliptic linear partial differential operators with constant coefficients.

References


Keywords: Differential operators; multi-quasielliptic operators; Gevrey-hypoellipticity; multi-anisotropic Gevrey spaces.

2010 Mathematics Subject Classification: 47E05; 35J62; 49N60.
A posteriori error analysis for solving the Navier-Stokes problem and convection-diffusion equations

Rahma Agroum

University of Tunis El Manar, Tunisia, and Université Pierre et Marie Curie, France

E-mail: rahmaagroum@gmail.com; agroum@ann.jussieu.fr

Abstract The following system models the stationary flow of a heated viscous incompressible fluid, we consider the finite element discretization of the Navier-Stokes problem coupled with convection-diffusion equations where both the viscosity and the diffusion coefficients depend on the temperature with boundary conditions which involve the velocity and the temperature. Note that, most often the realistic coupling of Navier-Stokes equations with the heat equation relies on two arguments:
1) the heat and more precisely the temperature is convected by the fluid,
2) the viscosity of the fluid depends on the temperature.

References


Keywords: Navier–Stokes problem; convection-diffusion equations; flow of heat.

2010 Mathematics Subject Classification: 35Q30; 76R05; 80A20.
Thermoelectric properties of sulvanites from first principles calculations

F. Litimein, N. Ben Bellil, H. Khachai, A. Yakoubi

Laboratoire d’Étude des Matériaux & Instrumentations Optiques, Faculté des Sciences Exactes, Université Djillali Liabès de Sidi Bel Abbès, Sidi Bel Abbès, Algeria

E-mail: f.litimein@yahoo.fr

Abstract The structural, electronic, thermoelectric and thermodynamic properties of sulvanite Cu$_2$TaCh$_3$ (Ch= S, Se) are investigated using the first principle calculations [1]. It is found that Cu$_2$TaS$_3$ and Cu$_2$TaSe$_3$ compounds are indirect semiconductors. Using EV−GGA approximation, the calculated band gaps are 2.102 eV and 1.798 eV, for Cu$_2$TaS$_3$ and Cu$_2$TaSe$_3$ respectively [2]. Semi-classic Boltzmann transport theory was then used to calculate the Seebeck coefficients, electrical conductivities and power factors of Cu$_2$TaCh$_3$ (Ch=S, Se)[3]. The temperature dependence of the thermoelectric transport properties of Cu$_2$TaCh$_3$ (Ch=S, Se) were also estimated. Moreover, thermodynamic properties (heat capacity and Debye temperature) as well as the thermal conductivity in a temperature range of 0−1000K are determined [4].

References


Keywords: sulvanite, DFT, EV−GGA, thermoelectric properties, thermodynamic properties.

2010 Mathematics Subject Classification: 35Q79; 35Q20.
On convexity for energy decay rates of a viscoelastic equation with a dynamic boundary and nonlinear delay term in the nonlinear internal feedback

Mounir Bahlil

Department of Mathematics, Mascara University, Mascara 29000, Algeria

E-mail: bahlilmounir@yahoo.fr

Abstract In this paper we consider the weak viscoelastic wave equation with a delay term in the nonlinear internal feedback

\[ u_{tt}(x, t) - \Delta u(x, t) + \alpha(t) \int_0^t h(t - s) \Delta u(x, s) ds + \mu_1 g_1(u_t(x, t)) + \mu_2 g_2(u_t(x, t - \tau)) = 0 \]

in a bounded domain, and prove a global existence result which depends on the behavior of both \( \alpha \) and \( h \) using the energy method combined with the Faedo–Galerkin procedure under a condition between the weight of the delay term in the feedback and the weight of the term without delay. Furthermore, we study the asymptotic behavior of solutions using a perturbed energy method.

References


Keywords: Weak viscoelastic equation; delay term; internal feedback, general decay rate.

2010 Mathematics Subject Classification: 35B40; 35L70.
Existence of minimal and maximal solutions for quasilinear elliptic equation
with nonlocal boundary conditions on time-scales

Mohammed Nehari¹, Mohammed Derhab²

¹Department of Mathematics, University Abou-Bekr Belkaid Tlemcen, Tlemcen 13000, Algeria
²Department of Mathematics, University Ibn Khaldoun Tiaret, Algeria

E-mail: nehari_72@yahoo.fr; derhab@yahoo.fr

Abstract The purpose of this work is the construction of minimal and maximal solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

\[
\begin{align*}
- (\varphi_p (u^\Delta))^\Delta &= f(x, u), \text{ in } (a, b)_T, \\
u(a) - a_0 u^\Delta(a) &= g_0(u), \\
u(\sigma(b)) + a_1 u^\Delta(\sigma(b)) &= g_1(u),
\end{align*}
\]

where \( p > 1, \varphi_p (y) = |y|^{p-2}y, (\varphi_p (u^\Delta))^\Delta \) is the one-dimensional \( p \)-Laplacian, \( f : [a, b]_T \times \mathbb{R} \rightarrow \mathbb{R} \) is a rd-continuous function, \( g_i : C_{rd}([a, b]_T) \times C_{rd}([a, b]_T) \rightarrow \mathbb{R} \) (\( i = 0 \) and \( 1 \)) are rd-continuous and \( a_0 \) and \( a_1 \) are a positive real numbers.

References


Keywords: Quasilinear elliptic equation; Time-Scale; Nonlocal boundary conditions; upper and lower solutions; monotone iterative technique.

2010 Mathematics Subject Classification: 34B10; 34B15.
A semilinear parabolic problem with singular term at the boundary

B. Abdellaoui¹, K. Biroud² and A. Primo ³

¹² Tlemcen University, Algeria ² University of Autonoma, Spain

E-mail: ¹boumediene.abdellaoui@uam.es; ² kh_biroud@yahoo.fr; ana.primo@icmat.es

Abstract In this work we deal with a class of a nonlinear parabolic problems related to the a Hardy inequality with singular weight at the boundary, more precisely, we consider the problem

\[
\begin{cases}
    u_t - \Delta u = \lambda \frac{u^p}{d^s} & \text{in } \Omega_T = \Omega \times (0, T), \\
    u > 0 & \text{in } \Omega, \\
    u(x, 0) = u_0(x) > 0 & \text{in } \Omega, \\
    u = 0 & \text{on } \partial \Omega \times (0, T),
\end{cases}
\]

we prove that

1. If \( p < 1 \) and \( s = 2 \) then (187) has non distributional positive solution.
2. If \( p < 1 \) and \( s < 2 \), then (187) has a distributional positive solution for all \( \lambda > 0 \) and for all \( u_0 > 0 \).

Introduction

The aim of this work is to discuss the existence and nonexistence of positive solution the following parabolic problem

\[
\begin{cases}
    u_t - \Delta u = \lambda \frac{u^p}{d^s} & u \geq 0, \text{in } \Omega_T = \Omega \times (0, T), \\
    u = 0 & \text{on } \partial \Omega \times (0, T), \\
    u(x, 0) = u_0(x) & \text{in } \Omega,
\end{cases}
\]

where \( \Omega \) is a bounded regular domain of \( \mathbb{R}^N \) \( d(x) = d(x, \partial \Omega) \), \( p > 0, 0 < s \leq 2 \) and \( \lambda > 0 \) is a positive constant.

Problem (188) is related to the following Hardy inequality: for all \( \phi \in W^{1,2}_0(\Omega) \),

\[
\Lambda(\Omega) \int_{\Omega} |\phi|^2 d^2 \leq \int_{\Omega} |\nabla \phi|^2
\]

where \( 0 < \Lambda(\Omega) \leq \frac{1}{4} \). If \( \Omega \) is a bounded convex, then \( \Lambda(\Omega) = \frac{1}{4} \) and it is not achieved. If \( \Lambda(\Omega) < \frac{1}{4} \), then \( \Lambda(\Omega) \) is attained we refer to [2] for more details about the attainability of the Hardy constant.

Keywords: Semilinear parabolic problems, Singular Hardy potential, Weak Harnack inequality, Complete blow-up results.

2010 Mathematics Subject Classification: 35B05, 35K15, 35B40, 35K55, 35K65.
Main results

The main nonexistence result is the following.

**Theorem 47.** Assume that $0 < p < 1$, $0 \leq s = 2$ then problem (18) has non distributional solution with $u \in L^1(\Omega \times (0, T))$ and $f \equiv u^p \frac{\partial u}{\partial z} \in L^1(\Omega \times (0, T), d)$. 

**Proof.** Without loss of generality, we can assume that $\lambda = 1$, $u_0 \in L^\infty(\Omega)$. We argue by contradiction. Assume that problem (18) has very weak nonnegative solution $u$ then by strong maximum principle $u > 0$ in $\Omega \times (0, T)$ Let $u_n$ be unique solution to approximating problems, it clear that $u$ is super solution to the approximating problems, thus by comparison principle and using an induction argument it follows that $u_n \leq u_{n+1} \leq u$. Hence we get the existence of $\overline{u} \in L^1(\Omega \times (0, T))$ such that $u_n \rightarrow \overline{u}$ strongly in $L^1(\Omega \times (0, T))$, a contradiction with the complete blow-up result Theorem 2.1 (see [1]). Hence we conclude.

The main existence result is the following

**Theorem 48.** Assume that $0 < p < 1$ and $0 < s < 2$. Then for all $u_0 \in L^\infty(\Omega)$ problem (18) has positive distributional solution $u$.

**Proof.** Let $u_n$ the approximated problems and by using a good test function and passing to the limit we obtain the desired result.

Acknowledgments

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References


Qualitative analysis of second order functional evolution inclusions

Abdessalam Baliki¹, Mouffak Benchohra², Juan J. Nieto³

¹,² Laboratory of Mathematics, University of Sidi Bel-Abbes
PO Box 89, Sidi Bel-Abbes 22000, Algeria
³ Departamento de Análisis Matemático, Facultad de Matemáticas,
Universidad de Santiago de Compostela, 15782,
Santiago de Compostela, Spain

E-mail: ¹adsbaliki@yahoo.fr, ²benchohra@univ-sba.dz, ³juan.jose.nieto.roig@usc.es

Abstract We investigate the existence and attractivity of mild solutions on infinite intervals to second order semilinear evolution inclusion with infinite delay in a Banach space. The proofs of the main results are based on Bohnenblust-Karlin's fixed point theorem and the theory of evolution system.

Introduction

We are concerned with the following second-order evolution inclusions

\[
\begin{cases}
y''(t) - A(t)y(t) \in F(t, y_t), & t \in J := [0, \infty), \\
y_0 = \phi, & y'(0) = \bar{y},
\end{cases}
\]

(190)

where \( \{A(t)\}_{0 \leq t < +\infty} \) is a family of linear closed operators from \( E \) into \( E \) that generate an evolution system of operators \( \{U(t, s)\}_{0 \leq s \leq t < +\infty} \), \( F: J \times \mathcal{B} \to P(E) \) is a multivalued map with nonempty compact convex values and \( \phi \in \mathcal{B} \). \( \mathcal{B} \) is an abstract phase space to be specified later, \( \bar{y} \in E, \phi \in \mathcal{B} \) and \( (E, |\cdot|) \) a real separable Banach space.

For any continuous function \( y \) and any \( t \geq 0 \), we denote by \( y_t \) the element of \( \mathcal{B} \) defined by \( y_t(\theta) = y(t + \theta) \) for \( \theta \in (-\infty, 0] \). Here \( y_t(\cdot) \) represents the history of the state up to the present time \( t \). We assume that the histories \( y_t \) belong to \( \mathcal{B} \).

References


Keywords: Semilinear functional differential inclusions of second order; mild solution; fixed-point; evolution system; infinite delay; infinite interval; attractivity; semigroup.

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Identification of pollution term in the two phase Stefan problem

Amel Berhail

Department of Mathematics, Guelma University, Algeria

E-mail: berhail_amel@yahoo.fr

Abstract In this paper, we study the identification of unknown pollution term in the two-phase stefan problem when we use the sentinel theory introduced by J.L. Lions. We prove the existence of such sentinels by solving a problem of null controllability with contraint on the control and we give information for the unknown source.

References


Keywords: Stefan problem; free boundary problem; optimal control; sentinel theory

2010 Mathematics Subject Classification: 80A22; 35R35; 49J20; 65M32.
Numerical solution of optimal control problem for the Oskolkov system

Guzel Baybulatova¹, Marina Plekhanova², Pavel Davydov³

¹,²,³ Chelyabinsk State University, Russia ²South Ural State University, Russia

E-mail: ¹baybulatova_g.d@mail.ru; ²mariner79@mail.ru; ³davydov@csu.ru

Abstract  In this paper numerical solution is found for a nonlinear system which is not resolved with respect to the time derivative. This system models a flow of the dynamics of Kelvin – Foight viscoelastic fluid. Solutions of original system and conjugate system are constructed by means of the difference schemes method. The conditional gradient method is used to find a numerical solution of optimal control problem for the system.

Let $\Omega \subset \mathbb{R}^n$ be bounded domain with smooth boundary $\partial \Omega$, $T > 0$, $Q = \Omega \times [t_0, T]$, $t_0 < T$. Consider the optimal control problem

$$(1 - \chi \Delta)z_t = v \Delta z - (z \cdot \nabla)z - \nabla p, \quad (s, t) \in Q,$$  \hspace{1cm} (191)

$$\nabla \cdot z = 0, \quad (s, t) \in Q,$$  \hspace{1cm} (192)

$$z(s, t) = 0, \quad (s, t) \in \partial \Omega \times [t_0, T],$$  \hspace{1cm} (193)

$$z(s, t_0) = u(s), \quad s \in \Omega$$  \hspace{1cm} (194)

where $z = (v, w)$ is the unknown vector-function, $\nabla p$ is the pressure gradient, $\chi, v$ are constants, $u = (u_1, u_2)$ is the control function. The admissible controls set is given by the next inequality with constant $R$

$$\|u_1\|^2_{L^2(Q)} + \|u_2\|^2_{L^2(Q)} \leq R^2. \hspace{1cm} (195)$$

Main goal of the problem is minimizing of the terminal functional

$$J(z) = \frac{1}{2} \|v(s, T) - \tilde{v}(s)\|^2_{L^2(Q)} + \frac{1}{2} \|w(s, T) - \tilde{w}(s)\|^2_{L^2(Q)}$$  \hspace{1cm} (196)

where $(\tilde{v}, \tilde{w})$ is the given function.

This system is not resolved with respect to the time derivative. Such systems are of interest because they do not fit into the framework of classical mathematical physics, and therefore if requires specific research methods. The convergence of numerical solutions under certain conditions on the constats and the time step of the difference scheme for the linearized Oskolkov system with initial-boundary condition has been proven in [1]. Unique solvability theorem to initial boundary value problem for the linearized strongly degenerate Oskolkov system has been obtained in [2]. Second step is finding solution of conjugate problem with help methods from [3]. The last step is carried out by means of an iterative conditional gradient method.

Conditional gradient method consists of several steps

- Choose constants $v, \chi, T, R$ and initial control function

$$u_0 = (\sin^2 x \sin y \cos y, - \sin^2 y \sin x \cos x).$$

Keywords: numerical solution, nonlinear system, control problem, Oskolkov system, degenerate system, conditional gradient method

2010 Mathematics Subject Classification: 49M30; 49M05; 35M99.
Construct sequence with help formula

\[ u_{k+1} = u_k + \alpha_k (u_k - u_k) \]

where \( \alpha_k, u_k \) is defined by conjugate problem solution.

The numerical experiment includes the following values of the program variables \( N=50, M=100, h=0.064, \tau = 0.01, \chi = 1, \nu = 1 \). Obtained graphs and the values of the cost functional are presented in the table below.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( J(z) )</th>
<th>1 ( 10^{-2} )</th>
<th>2 ( 10^{-2} )</th>
<th>3 ( 10^{-3} )</th>
<th>4 ( 10^{-3} )</th>
<th>5 ( 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6.8 ( 10^{-4} )</td>
<td>2.6 ( 10^{-4} )</td>
<td>9.97 ( 10^{-5} )</td>
<td>3.85 ( 10^{-5} )</td>
<td>1.5 ( 10^{-5} )</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>5.87 ( 10^{-6} )</td>
<td>2.3 ( 10^{-6} )</td>
<td>9.23 ( 10^{-7} )</td>
<td>3.74 ( 10^{-7} )</td>
<td>1.52 ( 10^{-7} )</td>
<td></td>
</tr>
</tbody>
</table>

References


Developing an Intelligent Fire Detection System on the Ships

Alena I Guseva1, Galina F Malykhina2, Alexey V Militsin3, Artem S Nevelskii4

Peter the Great Saint-Petersburg Polytechnic University, Russia

E-mail: 1Alyonaguseva1993@mail.ru; 2g_f_malychina@mail.ru; 3ctsp@mail.ru; 4artich0list.ru

Abstract The paper describes the development of an intelligent fire alarm system on the ships. In addition, there is a brief description of all the development stages, such as creating a model of ship rooms, selection an optimal sensor location and design of the final fire detection system. To calculate the model we used the Navier-Stokes equations describing low-speed flows. The calculation of the optimal arrangement of sensors based on a genetic algorithm. Intelligent processing of results from multicriterial sensors is performed with the help of a neural network.

Formulation of the modeling task.

It is necessary to simulate a fire in a given room to obtain experimental data. For our purposes, the field model was the best solution. To calculate the movement of air currents caused by a fire, we will use the hydrodynamic model. It allows solving the Navier-Stokes equations. Below are the basic equations used in this model. The mass transfer equation:

\[
\frac{dp}{dt} + \nabla \rho u = \vec{n}_b
\]

where \( \vec{n}_b \) the rate of change in mass in the isolated volume caused by the evaporation of droplets and other factors, \( \rho \) Mass density, \( t \) time, \( u \) speed. Law of conservation of momentum:

\[
\frac{d}{dt} (pu) + \nabla p uu + \nabla p = pg + f_b + \nabla \tau_{ij}
\]

where \( uu \) second-order tensor, \( f_b \) external forces caused by friction with droplets of liquid and other factors, \( \tau_{ij} \) stress tensor, \( S_{ij} \) strain tensor:

\[
\tau_{ij} = \mu(2S_{ij} - \frac{2}{3}\delta(\nabla u)); S_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}
\]

\[
S_{ij} = \frac{1}{2} \left( \frac{du_i}{dx_i} + \frac{du_j}{dx_j} \right); i, j = 1, 2, 3,
\]

The energy transfer equation:

\[
\frac{d}{dt} (ph_s) + \nabla ph_s u = \frac{Dp}{Dt} + q'' - q''_b - \nabla q' + e
\]

where \( h_s \) apparent enthalpy, \( \frac{Dp}{Dt} = \frac{dp}{dt} + u \nabla p \) material derivative, \( q'' \) the rate of formation of heat per unit volume due to chemical reactions, \( q''_b \) rate of heat absorption due to evaporation, \( q' \) reflects heat fluxes due to thermal conductivity and radiation:

\[
q' = -k \nabla T - \sum_{a} h_{s,a} p D_a \nabla Y_a + q''
\]

Keywords: modeling a fire, FDS, genetic algorithm, neural network, fire detect, multicriterial sensors.

2010 Mathematics Subject Classification: 28B04, 49M04.
where $Y_\alpha$ mass fraction of gas component $\alpha$, $D_\alpha$ diffusion coefficient of the gas component $\alpha$, $k$ thermal conductivity. Equation of state:

$$p = \frac{\rho RT}{\bar{W}}$$

(202)

where $\bar{W}$ average molar mass of the gas mixture. In the approach of the low Mach number, the external pressure $p(x,y,z)$, depending on the location in space goes into

$$p(x, t) = \bar{\rho}(z, t) + \tilde{\rho}(x, t); \bar{\rho}(Z, t) = \rho TR \sum_\alpha \frac{Y_\alpha}{W_\alpha}$$

(203)

where $\tilde{\rho}(x, t)$ agitation pressure, $W_\alpha$ molar mass $\alpha$ of the gas mixture fraction. For the numerical solution of equations, an explicit predictor-corrector scheme of the second order of accuracy with respect to space and time is used.

**Solution the problem on a supercomputer.**

As a solution tool, we selected a FDS (Fire Dynamics Simulator) program. Several runs with different resource allocation configurations have been conducted to select the best ways of parallelizing such applications.

1. Use only the OpenMP. We will use OpenMP on one node of a networked cluster, with one computational grid. During the 48 hours it was modeled around 1 minute real-time fire that has not produced any concrete outcomes.

2. Only the MPI. Based on the cluster power provided to us, namely four nodes, each of which has two processors with twelve cores, it was decided to divide the model computational grids and 96 computational grids, which corresponds to 96 cores of the entire system. Over 48 hours was modeled almost 4 minutes of real time and obtain more detailed values.

According to the results, we conclude that the use of MPI library reduces the simulation in a several times.

**Use of simulation results.**

The simulation results are intended to find the method of early fire detection. For this purpose, it is necessary to solve the problems of the optimal arrangement of fire sensors in a controlled room and the algorithm for making a decision about the occurrence of a fire. To solve the problem of choosing the location, it is suggested to use the genetic algorithm. An algorithm for early detection of fire was developed based on the data of multicriteria sensors.

**References**

Penalization of mixed Koiter’s shell model

Nora Tabouche

Department of Mathematics, Guelma University, 24000 Guelma, Algeria

E-mail: tabmoufida@yahoo.fr

Abstract In this paper, we consider the mixed problem discretized by finite elements of Koiter’s shell model. Then we uncoupling the primal and dual unknowns by penalization. We also lead a posteriori analysis in order to optimize the penelity parameter.

References


Keywords: Koiter’s shell model; mixed formulation; posteriori analysis; penalization.

2010 Mathematics Subject Classification: 74K25; 34C60.
Estimation of the system balance

V. V. Menshikh\(^1\), O. V. Pyankov\(^2\)

\(^{1,2}\) Voronezh Institute of the Ministry of Internal Affairs, Russia

E-mail: \(^1\)menshikh@mail.ru; \(^2\)pyankov@vimvd.ru

Abstract  The possibility of studying systems on the basis of conflict theory is considered in the article. Interval estimates of the system balance taking into account the conflicting properties of its elements are suggested. The approach of a complex system integrated assessment in terms of time and weight parameters is developed. A numerical example of their use for analyzing the graph model of an information–analytical system is presented.

The system representation of modeled objects usually involves their description as a finite set of elements and the relationships between them. Among the indicators of such systems performance, pride of place goes to their balance, i.e. the absence of internal conflicts and compromises. Subject areas within which the system balance has been traditionally examined include psychology (persistence of small groups), biology (food pyramid), economics (transport networks and urban economy) [1]. The presence in these areas of a wide variety of elements and relations between them allows them to be referred to as complex systems and to identify two ways of the conflict system study: 1. The description of the elements interaction in a general way taking into account all relevant factors, the detection and investigation of conflicting parties, possible nature of their interaction, causes and mechanisms of conflict. 2. On the assumption that the cause and nature of the conflict are known to the parties, the selection of the main, from the researcher’s point of view, factor (in the last resort, 2 - 3 factors) and the construction of a model for assessing the significance of a priori factors and the results of the conflict. Some authors [2, 3, 4] examined such concepts as the degree and intensity of the conflict, as well as measures of their measurement. This made possible to talk about a compromise solution to the conflict associated with the selection and construction of various compromise schemes and about optimization related to the search of the introduced terms. At the same time, assessments of the system balance have been provided in general terms, i.e. they were not based and accordingly not taking into account the conflicting properties of individual elements. Taking into account conflict of individual elements gives an opportunity of a more careful approach to the system balance analysis and permits to propose effective measures to improve it. Defining relations prevailing over the others in the system allows to draw conclusions on the effectiveness of its functioning and to make recommendations on the application of certain control actions. To describe relationships between the elements we use the terminology of the conflict theory [5]. We will consider the two elements S1, S2, which form some system S with the general purpose W. In achieving this purpose, subsystems interact consistent with their local purposes W1, W2. We introduce the set S = (S1, S2), S2 = (S1, S2 ), S3 = ( S1, S2 ), S4 = (S1, S2 ), where Si is the absence in the environment S of subsystem Si, and S = (S1, S2) = φ . We call a cycle balanced, if the number of arcs ui with the sign “−” is even. Accordingly, the graph is balanced, if all the cycles of the graph are balanced. The theoretical analysis of the balance at the level of structural analysis is carried out by means of assessments imposed by Harari [1]: \(\square^- = \square - / \square = (\square - \square +)/\square, \square^+ = \square + / \square = (\square - \square -)/\square\), where \(\square\) - the number of all directed cycles (hereinafter dicycles), \(\square^+\) - the number of balanced dicycles, \(\square^-\) - the number of unbalanced dicycles. However, the nonsufficiency of graph balance assessment (conflict of the system elements) can be assumed only through the ratio of balanced and unbalanced cycles. The proposed estimation indicators being points, this makes the analysis difficult. In this regard, it may be suggested to use interval estimates which consist in the fact that

Keywords: Conflict theory; balanced systems; weighted parameters; system dynamics; balanced graphs.

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instead of point values, we choose some limit values $M_{k_{\text{min}}}$ level, $V_{\text{max}}$ level etc., in respect of which sets of slow, heavy elements are formed. In other words, these sets will include only those elements whose parameters values exceed the boundary in the case of calculating the maximum estimates or do not exceed it, in the case of calculating the minimum estimates. In general, the subject area and the nature of the research should determine the choice of limit values. Then, respectively, for interval evaluations, the expressions (16) and (17) take the following form:

1. Interval K-degree system dynamics $\square dk = \frac{|sD_{k_{\text{lev.}}}|}{|S|}$, where $sD_k$ - the set of graphs nodes for which $D_k \leq D_{k_{\text{lev.}}}$.

2. Interval K-degree system statics $\square sk = \frac{|sS_{k_{\text{lev.}}}|}{|S|}$, where $sS_k$ - the set of graphs nodes for which $S_k \geq S_{k_{\text{lev.}}}$.

3. Interval estimation of strong elements of the system: $\square V_{\text{max}} = \frac{|sV_{\text{max}_{\text{lev.}}}|}{|S|}$, where $sV_{\text{max}}$ - the set of system elements for which $V_{\text{max}} \geq V_{\text{max}_{\text{lev.}}}$.

4. Interval estimation of weak elements of the system: $\square V_{\text{min}} = \frac{|sV_{\text{min}_{\text{lev.}}}|}{|S|}$, where $sV_{\text{min}}$ - the set of system elements for which $V_{\text{min}} \leq V_{\text{min}_{\text{lev.}}}$.

5. Interval estimation of fast elements of the system: $\square B = \frac{|sB_{k_{\text{max}_{\text{lev.}}}}|}{|S|}$, where $sB_{k_{\text{max}}}$ - the set of elements for which $B_{k_{\text{max}}} \geq B_{k_{\text{max}_{\text{lev.}}}}$.

6. Interval estimation of slow elements of the system: $\square M = \frac{|sM_{k_{\text{min}_{\text{lev.}}}}|}{|S|}$, where $sM_{k_{\text{min}}}$ - the set of elements for which $M_{k_{\text{min}}} \leq M_{k_{\text{min}_{\text{lev.}}}}$.

Conclusion. The study of complex systems in order to identify contradictions in it and to prepare rational solutions for their reduction or elimination requires a thorough analysis of the important and significant interactions between elements of the system. The approach to solving this problem on the basis of the conflict theory and proposed in the article estimates permit to obtain not only qualitative, but quantitative characteristics of the balance of the whole system and its elements. The use of interval estimates allows more accurate balance analysis of complex systems and their components. The detection of conflicting elements can be used for the structural changes in the system.

References


Adaptive Measurements of the Motion Parameters Under Uncertainty of the Moon Rock Composition

Irina Kislitcyna\textsuperscript{1}, Galina Malykhina\textsuperscript{2}

\textsuperscript{1}Russian State Scientific Center for Robotics and Technical Cybernetics, Russia
\textsuperscript{2}Peter the Great Saint Petersburg Polytechnic University, Russia

E-mail: \textsuperscript{1}irina_kislitsyna@mail.ru; \textsuperscript{2}g_f_malychina@mail.ru

Abstract Method of the descent module motion rate measurement is suggested. Measurement is based on the registration of scattering photons, emitted by central photon source and detected by four photon detectors under uncertainty of soil. Application of radio altimeter on the descent module allows to adapt photon altimeter to the moon rock composition. Mathematical model of measuring system is created and dependence of the photon flux intensity on the altitude and the angle are obtained.

Introduction

For successful landing on the Moon's surface it is necessary to execute precise control of landing engine near surface to this effect we must get the accurate measurement of current altitude, speed and slope angle of the descent module relative to underlying surface.

In the space industry are used laser and radio-altimeters, multibeam systems measuring the Doppler velocity of the spacecraft, the star sensors, gyroscopes, accelerometers, sensors of the sun and moon image sensors, integrated optical systems and inertial navigation. Therefore altimeters based on infrared and video cameras can be difficult determining the motion of the apparatus. In connection with the sharp focus of the laser radiation surface irregularities can cause large errors in the measurement. Errors in the measurement of radio wave altimeter increases with decreasing height, which is not acceptable to solve the problem. When the jet landing on the lunar surface dust rises, so the use of systems that are sensitive to the transparency of the medium. The limiting factor in the use of radio in the commission of interplanetary flights is the impossibility of propagation of electromagnetic waves through the plasma working soft landing engines.

Measurements at the altitudes greater than 10 m, are usually performed by the radio altimeter, at lower altitudes, altimeter provides significant measurement error. At the low altitudes, it is advisable to develop measurements based on registration of the scattered photon radiation.

Photon instrument enables to measure altitude through the plasma of engines, to take into consideration the density of the underlying surface and ignore the dust layer located on the surface, to reduce the effect of surface roughness. Measuring inaccuracy of photon altimeter decreases at low altitudes, which is a necessary condition for engine landing control. Therefore, the landing system was proposed to use photonic altimeter.

Main results

The algorithm combines the information of two altimeters operating on different physical principles: photon altimeter and the radio altimeter. If the module approaches the surface, the radio altimeter errors increase and
decrease errors photon altimeter. In the range of heights from 20 to 10 m two altimeter operating simultaneously, the information of one of them depends on the composition of the soil. This makes it possible to carry out the adaptation of the photon altimeter measurement algorithm using the radio altimeter data. At altitudes from 10 m to 0 m only photon altimeter measurements are used.

The scattering of gamma quanta depends on the type of the underlying surface. Placed on the surface of the lunar soil particles are large fragments and sparse rocks, which are gradually covered by micro-craters size from proportion of micrometers to a few centimeters. Cover of friable material formed on the surface of the Moon as a result of bombardment. This cover is called regolith, and it consists of fragments of bedrock. As part of the regolith contains substances and minerals lunar rocks: plagioclase (solid solution albite NaAlSi$_3$O$_8$ and anorthite CaAl$_2$Si$_2$O$_8$), orthopyroxene (Mg,Fe)SiO$_3$, clinopyroxene (Ca,Mg,Fe)SiO$_3$, olivine (Mg,Fe)$_2$SiO$_4$, ilmenite (FeTiO$_3$) and spinel group minerals (FeCr$_2$O$_4$ – Fe$_2$TiO$_4$ – FeAl$_2$O$_4$). Among the rocks in the bedrock of the Moon isolated a series of glandular anorthosite, a series of intrusive magnesian containing MgO – 7...45 %, Al$_2$O$_3$ – 2...29 %, TiO$_2$ – 0.5 %, alkaline intrusive series containing TiO$_2$ – 0.5...5 %, FeO – 0.4...17 %, K – 0.3...0.5 %, SiO$_2$ – 65...75 %, and a series of rare-earth non-marine basalts, which includes Al$_2$O$_3$ – 15...24 %, FeO – 9...15 % The composition of the lunar soil has been studied well in the landing areas by research units as a result of expeditions to the moon. However, the landing must succeed in previously unexplored areas of the deviation from the estimated landing area. For this purpose it is necessary to adapt the measurement algorithm to the landing place.

It is proposed to use a neural network algorithm for measuring motion parameters. An important property of neural networks is their ability to learn and to generalize the acquired knowledge. Trained on a limited set of training samples, neural network sums up the accumulated information and generates the expected response with regard to data that is not involved in the learning process. The neural network may independently determine uninformative for the analysis of parameters and produce their screenings. Than the adaptive capacity of the system is, the more stable it will work in non-stationary environment. The possibility of parallel information processing using contemporary processors provide the promptitude of neural networks. Because of this ability significantly accelerate data processing is achieved for a large number of interneuron connections. It is proposed to use nonlinear autoregressive network with external inputs, denoted usually as NARX.

References


On the Convergence of Moving Average Processes

Badreddine Azzouzi

E.P.S.E.G Tlemcen, Algeria

E-mail: azzouzi81@yahoo.fr

Abstract  Let \( \{Y_i, -\infty < i < \infty\} \) be a doubly infinite sequence of identically distributed and \( \psi \)-mixing random variables with zero means and finite variance and \( \{a_i, -\infty < i < \infty\} \) an absolutely summable sequence of real numbers. In this paper, we prove the complete moment convergence of \( \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} a_i Y_i / n^\gamma; n \geq 1 \) under some suitable conditions.

References


Keywords: Complete moment convergence; Moving average; Dependent variable.

2010 Mathematics Subject Classification: 60F05; 62M10.
Cutting stress models in dry hard turning using Taguchi technique

Nabil Kribes¹, Brahim Fnides¹,², Hamdi Aouici³, Smail Boutabba², Mohamed Athmane Yallese¹

¹L.M.A.N.M., Guelma University, Algeria; ²CMP, FGM&GP, USTHB, Algeria; ³ENST, Dergana-Alger, Algeria; ⁴LMS. Guelma University, Algeria

E-mail: normila1@yahoo.fr

Abstract  The purpose of this work is to model cutting stress in machining X38CrMoV5-1 treated at 50 HRC. This steel is intended for hot work and free from tungsten on CrMoV basis. It is employed for the manufacture of moulds, module matrices of door for car and helicopter rotor blades. The tests of straight turning were carried out under dry conditions using multilayer coated carbide GC3015. Nine experimental runs based on an orthogonal array (L9) of the Taguchi method were performed to derive objective functions to be optimized within the experimental domain. The correlations between the cutting parameters and performance measures like cutting stress, were established by multiple linear regression models. The correlation coefficients found showed that the developed models are reliable and could be used effectively for predicting the responses within the domain of the cutting parameters. Highly significant parameters were determined by performing an analysis of variance (ANOVA) and response surface methodology (RSM). The results indicate that axial stress (Ka), radial stress (Kr) and tangential stress (Kt) get affected mostly by feed rate. Its contributions on Ka, Kr and Kt are (70.29; 51.96 and 74.75), respectively.

References


Keywords: Taguchi technique; hard turning; cutting stress; carbure.

2010 Mathematics Subject Classification: 03C98; 74A10.
Study of the physical properties of the rare earth materials

Amina Benkaddour, Yousra Benkaddour, Nadjat Benbellil, Imen Benkaddour

University of Djillali Liabes, Physics Department, 22000 Sidi-bel-Abbes, Algeria

E-mail: Benkadour.amina@yahoo.fr; Yousra.benkaddour@yahoo.fr; nn.nn.phy.chi@gmail.com; Imen.benkadour@yahoo.fr

Abstract The structural, and mechanical properties of three components binaries HoN, HoP and HoAs in the three phases: Rock-salt (B1), CsCl (B2) and zinc-blende (B3) have been investigated by using the full-potential Linearized muffin tin orbital (FP-LMTO) within density functional theory (DFT). We employed the local density approximation (LDA) for the exchange-correlation (XC) potential. We have first calculated the structural properties, in particular the equilibrium parameter $a_0$, the bulk modulus and its derivative. Ours results are in good agreement with other theoretical and experimental results found in literature. Also we have calculated the elastic constants C11, C12 and C44 of the Rock-salt (B1) structure that are systematically compared with others results.

References


Keywords: Rare earth materials; structural and mechanical properties; density

2010 Mathematics Subject Classification: 70J50; 05C42.
Approximate Controllability of Stochastic Bounds of Stationary Distribution of a Single Server Queue with Repeated Attempts and Two-Phase Service

Mohamed Boualem

Research Unit LaMOS (Modeling and Optimization of Systems), Faculty of Technology, University of Bejaia, 06000 Bejaia, Algeria

E-mail: robertt15dz@yahoo.fr

Abstract This paper aims to study the monotonicity properties and the stochastic comparability of some performance measures of a single server queue with repeated attempts and two-phase service. Particularly, we give insensitive bounds for the stationary distribution of the embedded Markov chain of the model under consideration. To highlight the different obtained theoretical results, numerical examples based on simulation are provided.

Introduction

Retrial queues are characterized by the phenomenon that an arriving customer who finds the server busy upon arrival is obliged to leave the service area and repeat his demand after some time. Between trials, a blocked customer who remains in a retrial group is said to be in orbit [1, 2, 3, 4, 5]. Queueing system with two-phase service has received a significant amount of attention of the researchers working in the area of queueing and reliability theory. Each type of service has its own significant impact on the real time system. In phase service queueing system, all arriving customers are served in phases; out of these some are essential phases of service provided by the server to all arriving customers whereas some are optional services provided by the server to the customers depending upon their choice.

In many congestion situations, it is very difficult to carry out the performance analysis of queueing systems with phase service. Therefore, a wide variety of techniques are used for providing the solution of queueing problems in different frameworks (supplementary variable, matrix geometric, etc.). However, qualitative properties of stochastic models constitute an important analytical basis for approximation methods. One of the important qualitative properties and approximation methods is monotonicity that can be studied using the stochastic ordering.

The paper addresses the problem of finding insensitive bounds for the stationary distribution of the embedded Markov chain of a single server classical retrial queue with repeated attempts and two-phase service. The analysis resorts mainly to the embedded Markov chain approach and stochastic ordering comparisons to study the monotonicity properties and the stochastic comparability for this model relative to the convex ordering [1, 2, 3, 4, 5]. Particularly, we give insensitive bounds for the stationary distribution of the embedded Markov chain of the model under consideration. Finally, numerical examples based on simulation are presented.

Mathematical formulation of the model

Consider a queueing system consisting of two phases of service and a single server who follows the customer in service when he passes from the first phase to the second. Customers arrive to the system according to a Poisson
There is a single server who provides a preliminary first essential service (FES service) denoted by $B_1$ to all arriving customers. As soon as the FES of a customer is completed, then the customer may leave the system with probability $q = 1 - p$ or may be provided with a second optional service (SOS service), denoted by $B_2$, with probability $p$. The service times follow general laws with probability distribution functions $B_k(x)$, Laplace-Stieltjes transforms $\beta_k(s)$ and $n$th moment $\beta^n_k(s)$, $k = 1, 2$, where the subindex $k = 1$ (respectively $k = 2$) denotes the FES (respectively the SOS). Primary customers finding the server free upon arrival automatically start their FES. However, if a primary customer finds the server busy (attending FES or SOS), then he joins the orbit in order to seek service again until he finds the server free. The time between two successive repeated attempts of each customer in orbit is assumed to be exponentially distributed with rate $\nu$.

Main results

For this model, we have established conditions under which the transition operators and the stationary distributions of two embedded Markov chains associated to both systems, with the same structure but with different parameters are comparable relative to the convex ordering. Moreover, the main result of this paper consists in giving insensitive stochastic bounds for the stationary distribution of the embedded Markov chain by using the partial information about the ageing concepts of the service time with distribution function $B_k(x)$, $k = 1, 2$. Finally, to highlight the different obtained theoretical results, numerical examples based on simulation are provided. In a general manner, the theoretical results obtained are confirmed by the simulation ones (a good agreement between the analytical results and those of simulation). Besides, if the traffic intensity is low then the stochastic bounds derived are a good approximation for the stationary probabilities of the considered model, whatever is the time distribution of the two-phase service $B_1$ and $B_2$ (NBUE or NWUE). Consequently, the performances of such a system (mean length queue, mean waiting time, ...) can be estimated by those of an $M_2/M_2/1$ retrial queue with priority customers.

References


Generalized method of moments estimators for linear dynamic panel data using stata

Abdelkader Debouche, Noura Biri
Guelma University, Algeria
E-mail: kader.prof@gmail.com

Abstract The analysis of Panel data is the subject of one of the most active and innovative bodies of literature in econometrics. This type of data requires specific technique to estimate their parameters. The Generalized Method of Moments (GMM) is the most important methods used especially when the cross-section dimension is large and the time-series dimension is small. This method can provide solution to the problem through simultaneity, reverse causality and omitted variable. Our paper will focus on Generalized Method of Moments (GMM) estimators for linear dynamic Panel data models, and their implementation using Stata.

References

On some density, elastic and electronic properties of rare-earth intermetallic type of compounds

Y. Benkaddour, A. Yakoubi, I. Benakddour, A.Benakddour

Physics Department, University of Sidi Bel-Abbes, Sidi Bel-Abbes, Algeria

E-mail: Yousra.benkaddour@yahoo.fr; yakoubi_aek@yahoo.fr; imen.benakddour@yahoo.fr; benkadour.amina@yahoo.fr

Abstract We have performed the density functional theory (DFT) based structural, elastic and electronic properties of the rare-earth intermetallic $R_2Ni_2Pb$ ($R = Ho, Lu$ and $Sm$) type of compounds. The calculations are carried using the full potential-linearized augmented plane wave (FP-LAPW) method within the framework of local density approximation (LDA). The estimated values of the equilibrium lattice constants are in close agreement with their corresponding experimental values. The elastic constants ($C_{ij}$) are also estimated to understand the mechanical properties and the structural stability of the material. The density of states and charge densities are estimated to understand the nature of the binding of the material.

References


Symmetry properties of linear fractional filtration equation with the Riesz potential

Nikita S. Belevtsov¹, Stanislav Yu. Lukashuk²

Ufa State Aviation Technical University, Russia

E-mail: ¹nikitabelewtsov@mail.ru; ²lsu@ugatu.su

Abstract  We study the symmetry properties of linear fractional filtration equation with the Riesz potential. Three main results were obtained. Firstly, the prolongation of the Lie point transformation group on the Riesz potential was constructed. Secondly, a generalization of the Leibniz rule for the Riesz potential was derived. These two results are necessary for investigation symmetry properties of equations with the Riesz potential. Thirdly, the Lie symmetry group for the linear filtration equation was found, which will help us to construct invariant solutions and conservation laws in the future.

Introduction

We consider the filtration of viscous fluid through the nonhomogeneous porous medium with fractional nonlocality in space. We assume that the fractional generalization of the Darcy’s law is valid:

$$v = -\frac{k}{\mu} \nabla (R^\alpha p), \ \alpha \in (0, 1).$$  \hspace{1cm} (204)

Here $k$ is the permeability of the porous medium, $\mu$ is the viscosity of the fluid, $v$ is the fluid velocity, $p$ is the pressure, and

$$R^\alpha p(t, x, y) = \frac{1}{\gamma(\alpha)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p(t, \mu, \nu)}{(x-\mu)^2 + (y-\nu)^2} d\mu d\nu, \ \gamma(\alpha) = 2^\alpha \pi^{\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\frac{2-\alpha}{2}\right)$$  \hspace{1cm} (205)

is the Riesz potential in the two-dimensional space [1]. From (204), using the continuity equation we obtain the linear fractional filtration equation

$$u_t = \Delta (R^\alpha u).$$  \hspace{1cm} (206)

We study the symmetry properties of the equation (206) in the two-dimensional space.

Main results

The following theorems have been proved.

Keywords: the Riesz potential; fractional filtration; group analysis; symmetry; prolongation formula; Leibniz rule.

2010 Mathematics Subject Classification: 35R11; 70G65.
Theorem 49. If \( f(x, y) \) and \( g(x, y) \) are analytical functions of their arguments, then the Leibniz rule for the Riesz potential has the following form

\[
R^\alpha(fg) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{m!n!} \frac{\partial^{m+n} g}{\partial x^m \partial y^n} L_2^{m+n} f,
\]

where

\[
L_2^{m+n} f = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{\infty} \frac{(\mu - x)^m (\nu - y)^n f}{[(\mu - x)^2 + (\nu - y)^2]^{\frac{\alpha}{2}}} d\mu d\nu.
\]

We consider one-parameter group of point transformations \[2\]

\[
\bar{x} = x + a\xi_1(x, y, u) + o(a), \quad \bar{y} = y + a\xi_2(x, y, u) + o(a), \quad \bar{u} = u + a\eta(x, y, u) + o(a). \tag{207}
\]

Theorem 50. The transformation \[207\] implies infinitesimal transformation of the Riesz potential \[205\] of order \( \alpha \) of the function \( u(x, y) \) can be written in the form

\[
R^\alpha \tilde{u}(\bar{x}, \bar{y}) = R^\alpha u(x, y) + a\zeta_\alpha(x, y, u(x, y)),
\]

where \( \zeta_\alpha(x, y, u(x, y)) \) is given by the prolongation formula

\[
\zeta_\alpha(x, y, u(x, y)) = R^\alpha (\eta - \xi^1 u_x - \xi^2 u_y) + \xi^1 D_x R^\alpha u + \xi^2 D_y R^\alpha u.
\]

Theorem 51. The two-dimensional fractional diffusion equation \[206\] with the Riesz potential has the following linearly autonomous symmetries:

\[
\begin{align*}
X_1 &= \frac{\partial}{\partial x}, \quad X_2 = \frac{\partial}{\partial y}, \quad X_3 = \frac{\partial}{\partial t}, \quad X_4 = u \frac{\partial}{\partial u}, \quad X_5 = y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y}, \quad X_6 = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + (2 - \alpha) t \frac{\partial}{\partial t}, \\
X_\infty &= g(t, x, y) \frac{\partial}{\partial u},
\end{align*}
\]

where \( g(t, x, y) \) is any solution of equation

\[
D_t g = D_x^2 R^\alpha g + D_y^2 R^\alpha g.
\]

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References


On inverse problem for the parabolic equation with the initial condition specified on an inclined line

Ruslan Feshchenko

1 P. N. Lebedev Physical Institute of RAS, 53 Leninski Prospect, Moscow, 119991, Russia

E-mail: rusl@sci.lebedev.ru

Abstract We consider the inverse problem for 2D parabolic wave equation in case when the initial conditions are specified in an inclined line. It follows that extracting the initial conditions based on the image is not always possible but when a solution exists it is unique. Practical methods for the inverse problem solution are considered. The ability to solve the inverse problem for the inclined line or plane (in 3D case) is critically important for the coherent X-ray imaging techniques utilizing modern X-ray sources such free electron lasers and high harmonics.

Introduction

The parabolic wave equation (PWE) proposed more than 60 years ago [4] is widely used in the conventional and X-ray optics to describe the propagation of paraxial or quasi-paraxial electromagnetic [6] and acoustics [5] beams in free space as well as in inhomogeneous media. This versatile nature of PWE encourages us to search for new applications of it and new methods of its solution.

One of the underutilized mathematical properties of the PWE is a possibility to express the field amplitude in a portion of free space through the initial values of the amplitude specified in an inclined line or plane (in 3D case) not necessary orthogonal to the beam propagation direction. While the solution in this case has already been reported before: see, for instance, [2, 3, 1, 7] for details, at least one unresolved problem has persisted. This is the extraction of amplitude values in the initial inclined line or plane using the field amplitude values in the image line or plane, respectively.

In this work we attempt to outline possible solutions of the inverse problem. We investigate the existence and uniqueness of its solution and discuss the practical way of extracting the initial values of field amplitude from often incomplete data about the field amplitude in the image line.

Main results

We begin with the 2D PWE and assume for simplicity that the wave number \( k = 1/2 \)

\[
\frac{i}{\partial z} u + \frac{\partial^2 u}{\partial x^2} = 0. \tag{208}
\]

At the inclined line defined as \( x = x_0 - \tan \theta z \), where \( \theta \) is the inclination angle, initial conditions \( u_0 \) are specified and the solution of (208) is then

\[
u(x, z) = \frac{x \cos \theta - z \sin \theta}{2\sqrt{\pi i}} \int_{-\infty}^{\infty} \frac{u_0(s)}{(z + s \cos \theta)^{3/2}} \exp \left[ \frac{i(x + s \sin \theta)^2}{4(z + s \cos \theta)} \right] ds. \tag{209}
\]

Keywords: parabolic wave equation, coherent X-ray imaging, inverse problem, free electron laser.

2010 Mathematics Subject Classification: 35K20, 44A10, 45Bxx, 45Qxx.
It is possible to demonstrate that the inverse problem can be reduced to the solution of the following integral equation of the Fredholm second type for $u_0(z'')$

$$u_0(z'') = -\frac{i}{\pi} \int_{-\infty}^{\infty} \frac{u_0(\zeta)}{\sqrt{z''-\zeta}} d\zeta - \frac{1}{\sqrt{-\pi i}} \int_{0}^{\infty} \frac{u(x',z)}{\sqrt{z''-z'}} \exp \left[ -\frac{ix'^2}{4(z-z'')} \right] dx', \quad (210)$$

where $u(x',z)$ is the field amplitude in the image line. The equation (210) is a singular Fredholm equation and from the general theory it is known that it has a solution if and only if its right side is a sectionally analytical function and such a solution is unique [6]. The funding this solution in practice (if it exists) is a highly non-trivial task as the inverse problem is ill-posed. However in two special cases when it is a priori known that the initial amplitude is a purely real or imaginary function the solution is equal to the real or imaginary part of the right side of (210), respectively. In the general case the solution should be found using some type of iterative method.

Acknowledgments

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References


Approximate symmetries and invariant solutions of nonlinear fractional diffusion-wave equation

Regina D. Saburova¹, Stanislav Yu. Lukashchuk²

Ufa State Aviation Technical University, Russia

E-mail: ¹saburova-95@mail.ru; ²lsu@ugatu.su

Abstract  Time–fractional diffusion–wave equation with the Riemann–Liouville fractional derivative is considered for a particular case when the order of fractional differentiation is close to an integer number. An appropriate approximation for this equation by the differential equation of integer order with a small parameter is obtained. Full group classification with respect to approximate Lie point symmetries for this approximate equation is performed. Examples of using obtained approximate symmetries for constructing approximate invariant solutions are presented.

Introduction

Group analysis of differential equations is one of the powerful approach to investigating nonlinear differential equations, constructing their solutions and conservation laws. However, group classification results show that groups of point transformations admitted by fractional differential equations have much less symmetries than corresponding differential equations of integer order. In particular case when the order of fractional differentiation is close to an integer number, the problem of finding approximate symmetries and approximate solutions can be considered. In this case fractional derivatives can be expanded into the series with respect to the small parameter, and fractional differential equations can be approximated by equations with a small parameter. Then the theory of approximate transformation groups (see, e.g. [2]) can be used for finding approximate Lie point symmetries of such equations. For subdiffusion equation such approach was used in [1].

In this paper we use this approach to obtain an approximation of the nonlinear fractional differential equation

\[ D_t^\alpha u = (k(u)u_x)_x, \quad \alpha \in (1, 2). \tag{211} \]

Here \( D_t^\alpha u \) is the Riemann–Liouville time–fractional derivative \( \frac{\Gamma(\alpha)}{\Gamma(\alpha-1)} (\frac{d}{dt})^\alpha u(t) \). This equation is known as the diffusion–wave equation.

Main results

We consider (211) with \( \alpha = 2 - \varepsilon \), where \( \varepsilon \) is a small parameter (0 < \( \varepsilon \) ≪ 1). Then using the approach presented in [1], we approximate Eq. (211) with the accuracy \( o(\varepsilon) \) by the equation

\[ u_{tt} + \varepsilon \left[ \ln t + \gamma - \frac{3}{2} \right] u_{tt} - \frac{u_t}{t} + 2 \frac{u_t}{t} + 2 \sum_{n=1}^{\infty} \left( -1 \right)^n \frac{t^n}{n(n+2)!} D_t^{n+2} u \approx (k(u)u_x)_x. \tag{212} \]

Keywords: fractional derivative; diffusion–wave equation; small parameter; approximate Lie point symmetry; group classification; approximate invariant solution.

2010 Mathematics Subject Classification: 35B20; 35R11; 70G65.
We study symmetry properties of this equation using the theory of approximate Lie point transformation groups \[2\].

Full approximate group classification for Eq. \[(212)\] is performed with respect to the arbitrary function \(k(u) = k_0(u) + \varepsilon k_1(u)\). For several cases of the obtained classification, approximate invariant solutions are constructed using corresponding approximate symmetries.

**Example.** Let us consider Eq. \[(212)\] with \(k(u) = u^{-4}\). We obtain that this equation admits a nine-dimensional approximate Lie algebra spanned by the following symmetries

\[
\begin{align*}
X_1 &= \frac{\partial}{\partial x}, \\
X_2 &= (1 + \frac{\varepsilon}{2})t \frac{\partial}{\partial t} + x \frac{\partial}{\partial x}, \\
X_3 &= -2x \frac{\partial}{\partial x} + u \frac{\partial}{\partial u}, \\
X_4 &= t^2 \frac{\partial}{\partial t} + (1 - \varepsilon)tu \frac{\partial}{\partial u}, \\
X_5 &= \varepsilon \frac{\partial}{\partial t}, \\
X_6 &= \varepsilon \frac{\partial}{\partial x}, \\
X_7 &= \varepsilon t \frac{\partial}{\partial t} + \varepsilon xu \frac{\partial}{\partial x}, \\
X_8 &= -2\varepsilon x \frac{\partial}{\partial x} + \varepsilon u \frac{\partial}{\partial u}, \\
X_9 &= \varepsilon t^2 \frac{\partial}{\partial t} + \varepsilon tu \frac{\partial}{\partial u}. 
\end{align*}
\]

Note that symmetries \(X_1, X_2, X_3, X_4\) are stable.

Using the symmetry \(X_4\), we obtain the following approximate invariant solution to Eq. \[(212)\]:

\[
u(t,x) = \frac{(1 - \varepsilon \ln t)t}{(C_1 + C_2)^{1/3}}. \tag{213}\]

Here \(C_1 = C_1(\varepsilon), C_2 = C_2(\varepsilon)\) are arbitrary constants for a given \(\varepsilon\).

Solution \[(213)\] is also can be considered as an approximate solution of the initial diffusion-wave equation \[(211)\] with \(k(u) = u^{-4}\).

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About one dynamic system, characterizing free oscillations taking into account the variable heredity

R.I. Parovik

1 Institute of Cosmophysical Research and Radio Wave Propagation FEB RAS, Russia
2 Vitus Bering Kamchatka State University, Russia
E-mail: romaparovik@gmail.com

Abstract The mathematical model of the hereditary dynamic system is proposed, which describes free oscillations with allowance for the variable memory. Variable memory is reflected in the model in the form of derivatives of fractional order variables. Put the corresponding Cauchy problem, which was solved by the theory of finite-difference schemes. The constructed scheme was investigated for stability and convergence, the results of the research are formulated in the form of theorems.

Introduction

Dynamic systems with account for heredity were first mentioned in the work of the Italian mathematician Vito Volterra [1]. Heredity is characterized by the dependence of the current state of the system on previous states; therefore, it is sometimes called memory. Heredity is described by means of integro-differential equations with kernels, which are called memory functions. Memory functions can be different functions, but the power function of memory is of interest, since they are characteristic for fractal media. Therefore, fractal dynamical systems are a special case of hereditary dynamical systems. In the simulation of fractal dynamical systems, it is convenient from the integro-differential equations to proceed to equations with derivatives of fractional orders [2]. The orders of the derivatives are related to the fractal dimension of the medium and, in the general case, can depend on the time [3],[4].

In this paper we consider the Cauchy problem:

\[ \partial_{0t}^{\beta(t)} x(\eta) + \lambda \partial_{0t}^{\gamma(t)} x(\eta) = 0, \quad x(0) = \alpha_1, \quad \dot{x}(0) = \alpha_2, \]

where \( \partial_{0t}^{\beta(t)} x(\eta) = \frac{1}{\Gamma(2 - \beta(t))} \int_0^t \frac{\dot{x}(\eta)}{(t-\eta)^{\beta(t)-1}} d\eta \) and \( \partial_{0t}^{\gamma(t)} x(\eta) = \frac{1}{\Gamma(1 - \gamma(t))} \int_0^t \frac{\dot{x}(\eta)}{(t-\eta)^{\gamma(t)}} d\eta \) - fractional derivatives of orders \( 1 < \beta(t) < 2, 0 < \gamma(t) < 1, \dot{x}(t) = \frac{dx(t)}{dt}, t \in [0, T] \) - simulation time, \( \lambda \) - positive coefficient responsible for friction, \( \alpha_1 \) and \( \alpha_2 \) - given constants.

The hereditary system (1) describes free oscillations with memory. The solution of this system can be obtained with the numerical methods - the theory of finite-difference schemes [3],[4]. According to the procedure of paper [4], an explicit finite-difference scheme was constructed. Next, we give a series of auxiliary lemmas for the proof of the following key theorem.

Keywords: memory; model; stability; convergence; finite-difference scheme; Cauchy problem; derivative of fractional variable order

2010 Mathematics Subject Classification: 45J05; 26A33; 34A08; 34C10.
Main results

According to the procedure of paper [4], an explicit finite-difference scheme was constructed. Next, we give a series of auxiliary lemmas for the proof of the following key theorem.

**Theorem 52.** The explicit finite-difference scheme is stable and converges with the first order if the following condition is satisfied:

\[
\tau \leq \tau_0 = \min \left( 1, \frac{2 \Gamma \left( 2 - \gamma \left( t_j \right) \right)}{\lambda \Gamma \left( 3 - \beta \left( t_j \right) \right)} \right), j = 1, \ldots, N - 1,
\]

where \( N \) – number of points in the scheme.

Further, in the work the calculated curves were constructed depending on various values of the control parameters taking into account the ratio (2) and without taking it into account. It was confirmed that the stability of the circuit is fulfilled under condition (2). Further investigation of the Cauchy problem (1) is connected with the consideration of the nonlinear right-hand side in the original equation.

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References


Models of Two-phase Filtration in Fractal Porous Media

Ildar N. Abdulin\(^1\), Vitaliy A. Baikov\(^2\)

\(^1\) Ufa State Aviation Technical University, Russia  \(^2\) Ufa State Aviation Technical University, Russia

E-mail: \(^1\)nakiullowich@mail.ru; \(^2\)baikov@ufanipi.ru

Abstract  Classical models of two-phase filtration are generalized to fractal media by using power laws for geometrical properties of porous media (porosity and absolute permeability). Some general and self-similar solutions of the obtained models are constructed and examined. The ordinary differential equations for self-similar solutions are solved numerically.

Introduction  The mass distribution in fractal media can be described by the relation

\[ M \sim L^D, \]  

introduced by Mandelbrot\(^[1]\). Here \( M \) is the mass, \( L \) is the characteristic size of spatial domain, \( D \) is the Hausdorff dimension. From this relation, the density of the fractal object can be estimated as

\[ \rho \sim L^{D-E}, \]  

where \( E \) is the euclidean dimension. In general, density \( \rho \) is a stochastic function, we consider smoothed density \( \tilde{\rho}(r) \) that satisfy the relation

\[ \tilde{\rho}(r) \sim r^{D-E} \]  

or

\[ \tilde{\rho}(r) = \rho^* r^{D-E}, \quad \rho^* \equiv const. \]  

Applying the same method to porosity and absolute permeability of the fractal porous media\(^[2]\), one can get the following power laws:

\[ \tilde{m}(r) = m^* r^{D_m-E}, \quad m^* \equiv const, \]
\[ \tilde{k}(r) = k^* r^{D_k-E}, \quad k^* \equiv const, \]  

where \( m^*, k^*, D_m, D_k \) can be obtained from experimental data.
We generalize the classical two-phase filtration models to fractal media case by using these geometrical properties expressions in principal equations.

Keywords: two-phase flows, fractal media, power law relation.

2010 Mathematics Subject Classification: 76T10; 35C06; 35Q35; 37F99.
Main results

The principal equations of filtration theory are Darcy’s law and the mass conservation equation. The specific form of coefficients $m, k$ in these equations allows to model the fractal properties of the medium.

As an example, let us consider a generalization of Buckley-Leverett model [3] which describes filtration without discontinuities of saturation $s(t,x)$. Determining the movement of fluids is equivalent to solving partial differential equation in particular case:

$$\frac{\partial s}{\partial t} - \frac{F'(s)}{m^* x^{D_m-1}} \frac{\partial s}{\partial x} = 0, \quad F(s) = \frac{f_1(s)}{f_1(s) + \mu_0 f_2(s)} , \quad s(0, x) = \psi(x). \quad (221)$$

This model reduces to the classic case if $D_m = 1$.

General solution for this equation is obtained in parametric form:

$$s = \psi \left(D_m \theta^{1/D_m}\right), \quad \frac{x^{D_m}}{D_m} = \frac{t F'(s)}{m^*} + \theta. \quad (222)$$

For small enough pressure gradient, the distribution of saturation is only affected by the Hausdorff dimension – as it increases, the filtration speed decreases.

The next considered model is the generalized model of capillary imbibition [3], which describes absorption of fluid into a cylindrical sample. The differential equation for this case have the form

$$\frac{\partial s}{\partial t} + \frac{A}{x^{D_m-1}} \frac{\partial}{\partial x} \left(x^{D_{k-1}} x^a \frac{\partial}{\partial x} \left(x^{(D_m - D_k)/2} S^\beta \right) \right) = 0, \quad (223)$$

$$A, a, \beta \equiv const.$$ It allows automodel substitution

$$s(t, x) = t' S \left(\frac{x}{\sqrt{t'}}\right), \quad r = \frac{D_m - D_k}{4(a + \beta - 1)}. \quad (224)$$

As a result, one ordinary differential equation is obtained, which we solved numerically. Here the sum of Hausdorff dimensions has a large impact on the saturation distribution. We have also generalized other classical two-phase filtration models using similar methods.

The Rapoport-Lis model can also be generalized. The ODE for self-similar solution solutions were have also solved numerically.

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References


Stochastic bounds of a single server retrial queue two-way communication and multiple types of outgoing calls

Lala Maghnia Alem¹, Mohamed Boualem², Djamil Aissani³

¹ Research Unit LaMOS, University of Bejaia, Department of Mathematics, University of Bouira, Algeria
²,³ Research Unit LaMOS, University of Bejaia, Algeria

E-mail: ¹alem.nanou@yahoo.fr; ²robertt15dz@yahoo.fr; ³lamos.bejaia@hotmail.com

Abstract In this work the monotonicity properties of the embedded Markov chain of the M/G/1 retrial queue with two way communication and multiple types of outgoing calls is investigated via stochastic comparison approach. This approach is developed for comparing a non-Markov process to Markov Process with many possible stochastic ordering [3]. We show the monotonicity of the transition operator of the embedded Markov chain relative to the strong stochastic ordering and convex ordering, as well as the comparability of two transition operators.

Model description

The model considered in this work comes from Sakurai and Phung–Duc (2015)[1], where a single server retrial queues with two-way communication and n types of outgoing calls is considered. Primary incoming customers arrive according to a Poisson process with rate λ. Upon arrival, if the server is available it is served immediately. If not, incoming call joins the orbit and repeats its request after an exponentially distributed time with rate µ. If the server is free it makes an outgoing call of type i in an exponentially distributed time with rate αi (i = 2, 3, ..., n + 1).

We assume that the two types of calls (incoming calls and outgoing calls) receive different service times. Indeed, the service times of incoming calls and an outgoing call of type i (i = 1, 2, ..., n) are characterized by the distribution functions Bl(x) and B(i+1)(x) respectively. The Laplace–Stieltjes transform and the kth moment of Bl(x) by βl(s) and βk,l for l = 1, 2, ..., n, n + 1.

Embedded Markov chain

We consider the embedded Markov chain representing the number of customers in the orbit at the service completion epoch of either an incoming call or an outgoing call. It is easy to see that the one-step transition probabilities of this Markov chain are given as follows:

\[ p_{i,i-1} = \frac{\mu}{\lambda + \mu}\sum_{m=2}^{i+1} a_m + i\mu m_{i-1}, \quad i \geq 1; \]

\[ p_{i,j} = \begin{cases} \frac{\lambda}{\lambda + \mu} k_1^{i-j} + \frac{\mu}{\lambda + \mu} k_2^{i-j} + \frac{\mu}{\lambda + \mu} k_3^{i-j}, & 0 \leq i \leq j; \\ 0, & i - 1 \geq 2. \end{cases} \]

Where,

\[ k_l^j = \int_0^\infty e^{-\lambda x} (\lambda x)^{l-1} dB_l(x), \quad l = 1, 2, ..., n + 1, \quad j \in \mathbb{Z}_+, \]

Keywords: Retrial queues; two-way communication; embedded markov chain; monotonicity; stochastic comparison.

2010 Mathematics Subject Classification : 90B22; 35R60.
Main results

We consider two $M/G/1$ retrial queues with two way communication and $n$ types of arbitrarily distributed outgoing calls $\Sigma_1$ and $\Sigma_2$ with parameters $\lambda^{(i)}, \mu^{(i)}, a^{(i)}, B^{(i)}(x), B^{(i)}_l(x), k^{(i)}_l,$ and $\pi^{(i)}_n$ (the stationary distribution in $\Sigma_i$), $i = 1, 2$.

Lemma 53. if $\lambda^{(1)} \leq \lambda^{(2)}$, $B^{(1)}_1 \leq_B B^{(2)}_1$ and $B^{(1)}_l \leq_B B^{(2)}_l$ then $\{k^{(1)}_n\} \leq_B \{k^{(2)}_n\}$, so = $(st, v)

Monotonicity properties of the embedded Markov chain

Let $\tau$ be the transition operator of an embedded Markov chain, which associates to every distribution $p = (p_n)_{n=0}^\infty$, a distribution $\tau p = q = (q_m)_{m=0}^\infty$ telle que $q_m = \sum_{n=0}^\infty p_n p_{nm}$ (where $p_{n,m}$ are one-step transition probabilities of the considered chain). We add the transition operators $\tau^{(1)}$ and $\tau^{(2)}$ to models $\Sigma_1$ and $\Sigma_2$, respectively.

Theorem 54. Under the condition $\sup_{2 \leq l \leq n+1} B_l \leq_B \frac{1}{n+1} \sum_{l=2}^{n+1} \alpha_l \mu$, the transition operator $\tau$ is monotone with respect to the stochastic order $\leq_{st}$, i.e. for any two distribution $p^{(1)}$ and $p^{(2)}$, the inequality

$\quad p^{(1)} \leq_{st} p^{(2)} \Rightarrow \tau p^{(1)} \leq_{st} \tau p^{(2)}$.

Proof. The operator $\tau$ is monotone with respect to $\leq_{st}$ if and only if [2]

$\quad \overline{p}_{n-1,m} \leq \overline{p}_{n,m}, \forall n, m,$

(225)

$\quad \overline{p}_{n,m} - \overline{p}_{n-1,m} = \frac{n+1}{(\lambda + \sum_{l=2}^{n+1} \alpha_l n \mu)(\lambda + \sum_{l=2}^{n+1} \alpha_l (n-1) \mu)} [k^{m-n+1}_1 - k^{m-n+1}_2]$

$\quad + \frac{\lambda}{\lambda + \sum_{l=2}^{n+1} \alpha_l n \mu} k^{m-n}_1 + \frac{(n-1) \mu}{\lambda + \sum_{l=2}^{n+1} \alpha_l (n-1) \mu} k^{m-n+1}_1 + \frac{\sum_{l=2}^{n+1} \alpha_l}{\lambda + \sum_{l=2}^{n+1} \alpha_l n \mu} k^{m-n}_2 \geq 0.$

Consequently, since $\sup_{2 \leq l \leq n+1} B_l \leq_B \frac{1}{n+1} \sum_{l=2}^{n+1} \alpha_l \mu$, then, the inequality (225) is verified.

Finally, the operator $\tau$ is monotone with respect to $\leq_{st}$.

References


A Bayesian approach to bandwidth selection in nonparametric kernel regression estimation for multivariate count data

Lamia Djerroud\textsuperscript{1}, Smail Adjabi\textsuperscript{2}

\textsuperscript{1}Research Unit LaMOS, University of Bejaia, Algeria \hspace{1em} \textsuperscript{2}Research Unit LaMOS, University of Bejaia, Algeria

E-mail: \textsuperscript{1}djerroudlamia@live.fr; \textsuperscript{2}adjabi@hotmail.com

Abstract Nonparametric regression is an important tool for exploring the unknown relationship between a response variable and a set of explanatory variables also known as regressors. This paper introduces the associated discrete kernel for multivariate nonparametric count regression estimation. We propose a Bayesian approach based upon likelihood cross-validation and a Monte Carlo Markov chain (MCMC) method for deriving the global optimal bandwidths. Through simulation data the comparative study of the Bayesian approach and cross-validation technique are presented.

Introduction

Recently, the discrete associated kernel is proposed for both probability mass and count regression functions (pmf and crf) in nonparametric estimation, see; e.g., [1], [2] and [3]. A famous example of an associated kernel is the discrete triangular kernel of [1], which was recently improved by [4]. The second competitive family is called the standard discrete kernel or first order. We have the binomial, Poisson, and negative binomial kernels; see [3] for more detail. Similar to continuous kernel method, the main issues of the discrete kernel method is the bandwidth selection procedure. Note that the classical method such as the cross validation technique and the Bayesian approach have been largely investigated for bandwidth choice in pmf and crf estimation by the associated discrete kernel method; see [1], [2], [3] and [5]. The multivariate discrete case has been far less investigated in comparison with univariate discrete case. The goal of this paper is we propose a nonparametric estimator for crf of multivariate data by using the product of univariate discrete kernels. In the same spirit that [5], we develop the Bayesian approach for estimating the bandwidth diagonal matrix $H = \text{Diag}(h_j)_{j=1,...,d}$, where $d$ is the dimension of the count data.

Multivariate discrete associated kernel for count regression function

Let an independent and identically distributed (i.i.d.) sample of a sequence of pairs $(x_i, y_i)_{i=1,...,n}$ taking values in the space $\mathbb{N}^d \times \mathbb{R}$. The nonparametric regression in discrete multivariate case is given by:

$$y_i = m(x_i) + \epsilon_i, \quad (226)$$

where $y_i$ is the response variable, $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})^\top$ is the multivariate count explanatory variable and $\epsilon_i$ denotes an i.i.d. error term with mean 0 and variance $\sigma^2 < \infty$, for $i = 1, \ldots, n$ and $m : \mathbb{N}^d \rightarrow \mathbb{R}$ is the multivariate regression count function.

Keywords: Bayesian approach; Nonparametric count regression; Discrete kernel; Multivariate data; Cross-validation technique.

2010 Mathematics Subject Classification: 62G08; 62G05; 62H12.
The discrete multivariate version of the Nadaraya-Watson estimator can be adapted as follows (see [2] for discrete univariate case):

\[ \hat{m}(\mathbf{x}) = \frac{\sum_{i=1}^{n} \mathbf{K}_{\mathbf{x},h}(\mathbf{x}_i) y_i}{\sum_{i=1}^{n} \mathbf{K}_{\mathbf{x},h}(\mathbf{x}_i)}, \quad \mathbf{x} \in \mathbb{N}^d, \] (227)

where \( \mathbf{h} = (h_1, h_1, \ldots, h_d) \) is the vector of bandwidths and \( \mathbf{K}(\cdot) \) is the discrete multivariate associated kernel defined as a product of associated univariate discrete kernel such as:

\[ \mathbf{K}_{\mathbf{x},h}(\mathbf{x}_i) = \prod_{j=1}^{d} K_{x_j,h_j}(x_{ij}), \] (228)

where \( K_{x_{ij},h_j}^{(j)} \) for \( j = 1, \ldots, d \) is the univariate associated discrete kernel satisfying the following conditions

\( (H1) \lim_{h_j \to 0} E(K_{x_{ij},h_j}^{(j)}) = x_j \) and \( (H2) \lim_{h_j \to 0} Var(K_{x_{ij},h_j}^{(j)}) = 0, \) with \( K_{x_{ij},h_j}^{(j)} \) is the discrete random variable of \( K_{x_{ij},h_j}^{(j)} \) defined on \( S_{x_{ij}} \subseteq \mathbb{N}. \)

**Bayesian bandwidth diagonal matrix selection**

We propose a bayesian sampling approach for the bandwidth estimation for the NW estimator involving discrete types of regressors. For such an approach in the non-parametric regression, one can refer to [7] with continuous regressors and [6] with mixed types continuous and categorical data.

**References**


On Vibrations of Prestressed Electroelastic Bodies

Vladimir V. Dudarev$^{1,2}$, Roman M. Mnukhin$^1$, Rostislav R. Nedin$^{1,2}$, Alexander O. Vatulyan$^{1,2}$

$^1$Southern Federal University, Russia $^2$Southern Mathematical Institute, Russia

E-mail: dudarev_vv@mail.ru

Abstract The general statement of motion of an electroelastic body in the presence of an inhomogeneous initial stress-strain state (ISS) is presented. On its basis, problems of steady-state vibrations of a rod and a thin cylindrical disc are formulated. Solution of the direct problem on determination of displacement functions is realized numerically by means of the shooting method. An influence of level and structure of ISS on the frequency response functions and the resonant frequencies is analyzed. New approaches to solving the inverse problems on determination of ISS are proposed.

Introduction

Today, the study of prestress (PS) in elastic and electroelastic bodies is of great importance from point of view of mathematical modelling in mechanics. Such stress often arises in piezoelectric bodies as a result of manufacturing or action of latent loads. It should be noted that the highest concentration of PS is observed in a vicinity of different defects. Distribution of such PS is usually essentially inhomogeneous. As the most adequate models describing the behavior of prestressed objects one can mention the models obtained on the basis of the principle of imposing small deformation on a finite one. In this paper, we consider the model of electroelastic behaviour of a solid body in the presence of ISS described, for instance, in [1].

Main results

It should be noted that the acoustic sounding method is one of the most efficient methods for investigation of different properties of solids. On the basis of [2], using a rigorous mathematical technique, we have obtained two new boundary-value problems for a rod and a thin cylindrical disc. The problem of steady longitudinal vibrations of a piezoelectric rod has the form:

$$
\left(\left(E^* + \frac{d^{*2}}{c}u^{*}F\right)u^*F\right)' + \rho F\omega^2 u = 0, \quad (229)
$$

$$
u(0) = 0, \quad \left(\left(E^* + \frac{d^{*2}}{c}u^*F\right)u^*F\right)(l) = P, \quad (230)
$$

where $E^* = E\left(1 + u^0\right)^2 + \sigma_{11}^0$ is the effective elastic modulus, $d^* = d(1 + u^0)$ is the effective piezoelectric modulus, $E$ is the Young modulus, $d$ is the piezoelectric modulus, $c$ is the dielectric constant, $\sigma_{11}^0$ is the PS tensor component, $u^0$ is the component of the residual displacement vector, $u$ is the component of the incremental small displacements, $F$ is the cross-sectional area, $\rho$ is the rod density, $\omega$ is the vibration frequency, $l$ is the rod length.

Keywords: electroelasticity; initial stress-strain state; vibration; rod; disk; direct problem; inverse problem.

2010 Mathematics Subject Classification: 74B15; 74E05; 74H15.
rod is clamped by its left end. The vibrations are caused by the load applied at the right rod end. With the help of \((229)-(230)\) and specifying the laws of variation of \(E\) and \(d\) we may consider problems for functionally graded piezoelectric rods. Since \((229)\) is a second-order differential equation with variable coefficients with respect to the function \(u\), its solution in the general case can be obtained numerically by means of the shooting method. A uniaxial tension is considered as an example of ISS. We have conducted the analysis of vibration modes of the rod, the amplitude-frequency characteristic and the values of the resonance frequencies in dependence on the level and structure of ISS. Since there are models which do not consider residual deformations, we have performed the analysis of the variation of these characteristics separately without taking into account the value of \(u^0\).

In order to solve the inverse problem of reconstructing the law of variation of ISS according to the data from the amplitude-frequency characteristic measured in a certain range, we have used the approach from \(2\) based on a construction of an iterative process and solving the Fredholm integral equation of the first kind.

The problem of steady-state radial vibrations of the thin cylindrical disk made of piezoceramics PZT-4 is considered as a second example. End-face surfaces of the disk are electrodeposited. Vibrations are caused by applying a potential difference. One-dimensional formulation of the problem for an elastic field with respect to the radial displacement function is obtained within the framework of the generalized plane stress. ISS corresponding to the solution of the Lame problem is considered as a model example. The direct problem is reduced to the numerical solving of a system of two first-order differential equations. The analysis of variation of the main dynamic characteristics of the disk is carried out.

In order to solve the inverse problem of determination of the level of ISS according to the data from variation the resonance frequencym we have employed the approach from \(3\) based on the analysis of free vibrations of a disk in the presence and the absence of ISS.

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